A level Mathematics Practice Paper – Integration – Mark scheme

Question	Scheme	Marks
1(a)	$1 = A(3x-1)^2 + Bx(3x-1) + Cx$	B1
	$x \to 0 \qquad (1 = A)$	M1
	$x \to \frac{1}{3} \qquad 1 = \frac{1}{3}C \;\Rightarrow\; C = 3$ any two constants correct	A1
	Coefficients of x^2	
	$0 = 9A + 3B \;\Rightarrow\; B = -3$ all three constants correct	A1
		(4)
1(b)	(i) $\int \left(\frac{1}{x} - \frac{3}{3x-1} + \frac{3}{(3x-1)^2} \right) \mathrm{d}x$	
	$= \ln x - \frac{3}{3}\ln(3x-1) + \frac{3}{(-1)3}(3x-1)^{-1} \quad (+C)$	M1 A1ft A1ft
	$\left(= \ln x - \ln(3x-1) - \frac{1}{3x-1} \quad (+C) \right)$	
	(ii) $\int_1^2 \mathrm{f}(x)\,\mathrm{d}x = \left[\ln x - \ln(3x-1) - \frac{1}{3x-1} \right]_1^2$	
	$= \left(\ln 2 - \ln 5 - \frac{1}{5} \right) - \left(\ln 1 - \ln 2 - \frac{1}{2} \right)$	M1
	$= \ln \frac{2 \times 2}{5} + \ \ldots$	M1
	$= \frac{3}{10} + \ln\left(\frac{4}{5}\right)$	A1
		(6)
		(10 marks)

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Question	**Scheme**		**Marks**
2	$\int x\sin 2x\,\mathrm{d}x = -\dfrac{x\cos 2x}{2} + \int \dfrac{\cos 2x}{2}\,\mathrm{d}x$		M1 A1 A1
	$= \quad \ldots \quad + \dfrac{\sin 2x}{4}$		M1
	$\left[\ \ldots\ \right]_0^{\frac{\pi}{2}} = \dfrac{\pi}{4}$		M1 A1
			(6 marks)
3(a)	$\int \dfrac{1}{x^3}\ln x\,\mathrm{d}x, \quad \left\{\begin{array}{l} u = \ln x \quad \Rightarrow \quad \dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{1}{x} \\ \dfrac{\mathrm{d}v}{\mathrm{d}x} = x^{-3} \quad \Rightarrow \quad v = \dfrac{x^{-2}}{-2} = \dfrac{-1}{2x^2} \end{array}\right\}$		
	$= \underline{\dfrac{-1}{2x^2}\ln x} - \underline{\underline{\int \dfrac{-1}{2x^2}.\dfrac{1}{x}}}\,\mathrm{d}x$	In the form $\dfrac{\pm\lambda}{x^2}\ln x \pm \int \mu\,\dfrac{1}{x^2}.\dfrac{1}{x}$	M1
		$\underline{\dfrac{-1}{2x^2}\ln x}$ simplified or un-simplified.	$\underline{\text{A1}}$
		$\underline{\underline{-\int \dfrac{-1}{2x^2}.\dfrac{1}{x}}}$ simplified or un-simplified.	$\underline{\underline{\text{A1}}}$
	$\left\{= \dfrac{-1}{2x^2}\ln x + \dfrac{1}{2}\int \dfrac{1}{x^3}\,\mathrm{d}x\right\}$		
	$= -\dfrac{1}{2x^2}\ln x + \dfrac{1}{2}\left(-\dfrac{1}{2x^2}\right)\ \{+\,c\}$	$\pm\int \mu\,\dfrac{1}{x^2}.\dfrac{1}{x} \ \rightarrow \pm\beta x^{-2}.$	dM1
		Correct answer, with/without $+\,c$	A1
			(5)
3(b)	$\left\{\left[-\dfrac{1}{2x^2}\ln x - \dfrac{1}{4x^2}\right]_1^2\right\} = \left(-\dfrac{1}{2(2)^2}\ln 2 - \dfrac{1}{4(2)^2}\right) - \left(-\dfrac{1}{2(1)^2}\ln 1 - \dfrac{1}{4(1)^2}\right)$		
	Applies limits of 2 and 1 to their part (a) answer and subtracts the correct way round.		M1
	$= \dfrac{3}{16} - \dfrac{1}{8}\ln 2$ or $\dfrac{3}{16} - \ln 2^{\frac{1}{8}}$ or $\dfrac{1}{16}(3 - 2\ln 2)$, etc, or awrt 0.1 or equivalent.		A1
			(2)
			(7 marks)

Question	Scheme	Marks
4(a)	$\int x\sin 3x \, \mathrm{d}x = -\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x \, \{\mathrm{d}x\}$	M1 A1
	$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x \, \{+c\}$	A1
		(3)
4(b)	$\int x^2\cos 3x \, \mathrm{d}x = \frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x \, \{\mathrm{d}x\}$	M1 A1
	$= \frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right) \; \{+c\}$	A1 **isw**
	$\left\{= \frac{1}{3}x^2\sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x \; \{+c\}\right\}$	
	Ignore subsequent working	**(3)**
		(6 marks)
5(a)	$y = 4x - x\mathrm{e}^{\frac{1}{2}x}, \; x \geqslant 0$	
	$\left\{y = 0 \Rightarrow 4x - x\mathrm{e}^{\frac{1}{2}x} = 0 \Rightarrow x(4 - \mathrm{e}^{\frac{1}{2}x}) = 0 \Rightarrow\right\}$	
	$\mathrm{e}^{\frac{1}{2}x} = 4 \Rightarrow x_A = 4\ln 2$	M1 A1
		(2)
5(b)	$\left\{\int x\mathrm{e}^{\frac{1}{2}x}\,\mathrm{d}x\right\} = 2x\mathrm{e}^{\frac{1}{2}x} - \int 2\mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$	M1 A1
	$= 2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x} \; \{+c\}$	A1
		(3)
5(c)	$\left\{\int 4x\,\mathrm{d}x\right\} = 2x^2$	B1
	$\left\{\int_0^{4\ln 2} (4x - x\mathrm{e}^{\frac{1}{2}x})\,\mathrm{d}x\right\} = \left[2x^2 - \left(2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}\right)\right]_0^{4\ln 2 \text{ or } \ln 16 \text{ or their limits}}$	
	$= \left(2(4\ln 2)^2 - 2(4\ln 2)\mathrm{e}^{\frac{1}{2}(4\ln 2)} + 4\mathrm{e}^{\frac{1}{2}(4\ln 2)}\right) - \left(2(0)^2 - 2(0)\mathrm{e}^{\frac{1}{2}(0)} + 4\mathrm{e}^{\frac{1}{2}(0)}\right)$	M1
	$= \left(32(\ln 2)^2 - 32(\ln 2) + 16\right) - (4)$	
	$= 32(\ln 2)^2 - 32(\ln 2) + 12$	A1
		(3)
		(8 marks)

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Question	Scheme		Marks
6(a)	1.154701		B1 **cao**
			(1)
6(b)	Area $\approx \frac{1}{2} \times \frac{\pi}{6}$;$\times \underline{\left[1+2\left(1.035276 + \text{ their } 1.154701\right)+1.414214\right]}$		B1 <u>M1</u>
	$= \frac{\pi}{12} \times 6.794168 \; = \; 1.778709023... = 1.7787$ (4 dp)	1.7787 or awrt 1.7787	A1
			(3)
6(c)	$V = \pi \int_0^{\frac{\pi}{2}} \left(\sec\left(\frac{x}{2}\right)\right)^2 \mathrm{d}x$	For $\pi \int \left(\sec\left(\frac{x}{2}\right)\right)^2$. Ignore limits and $\mathrm{d}x$. Can be implied.	B1
	$= \{\pi\}\left[2\tan\left(\frac{x}{2}\right)\right]_0^{\frac{\pi}{2}}$	$\pm\lambda\tan\left(\frac{x}{2}\right)$	M1
		$2\tan\left(\frac{x}{2}\right)$ or equivalent	A1
	$= 2\pi$	2π	A1 **cao cso**
			(4)
			(8 marks)

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Question	Scheme	Marks
7(a)	$\dfrac{5}{(x-1)(3x+2)}=\dfrac{A}{x-1}+\dfrac{B}{3x+2}$	
	$5=A(3x+2)+B(x-1)$	
	$x\to 1 \qquad 5=5A \Rightarrow A=1$	M1 A1
	$x\to -\dfrac{2}{3} \qquad 5=-\dfrac{5}{3}B \Rightarrow B=-3$	A1
		(3)
7(b)	$\displaystyle\int \frac{5}{(x-1)(3x+2)}\,\mathrm{d}x=\int\left(\frac{1}{x-1}-\frac{3}{3x+2}\right)\mathrm{d}x$	
	$=\ln(x-1)-\ln(3x+2) \quad (+C)$ ft constants	M1 A1ft A1ft
		(3)
7(c)	$\displaystyle\int \frac{5}{(x-1)(3x+2)}\,\mathrm{d}x=\int\left(\frac{1}{y}\right)\mathrm{d}y$	M1
	$\ln(x-1)-\ln(3x+2)=\ln y \quad (+C)$	M1 A1
	$y=\dfrac{K(x-1)}{3x+2}$ depends on first two Ms in (c)	M1 dep
	Using $(2,8)$ $\quad 8=\dfrac{K}{8}$ depends on first two Ms in (c)	M1 dep
	$y=\dfrac{64(x-1)}{3x+2}$	A1
		(6)
		(12 marks)

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Question	Scheme	Marks
8(a)	$\dfrac{3y-4}{y(3y+2)} \equiv \dfrac{A}{y} + \dfrac{B}{(3y+2)} \Rightarrow 3y-4 = A(3y+2) + By$ $y = 0 \Rightarrow -4 = 2A \Rightarrow A = -2$ $y = -\frac{2}{3} \Rightarrow -6 = -\frac{2}{3}B \Rightarrow B = 9$	M1 A1 A1
	$\displaystyle\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y = \int \frac{-2}{y} + \frac{9}{(3y+2)}\,\mathrm{d}y$ $= -2\ln y + 3\ln(3y+2)\ \{+c\}$	M1 A1 ft A1 **cao**
		(6)
8(b)	$\left\{x = 4\sin^2\theta \Rightarrow\right\} \dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 8\sin\theta\cos\theta$ or $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 4\sin 2\theta$ or dx = 8sin$q$cos$q$dq	B1
	$\displaystyle\int\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\,.\,8\sin\theta\cos\theta\ \{\mathrm{d}\theta\}$ or $\displaystyle\int\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\,.\,4\sin 2\theta\ \{\mathrm{d}\theta\}$	M1
	$= \displaystyle\int \underline{\underline{\tan\theta}}\,.\,8\sin\theta\cos\theta\ \{\mathrm{d}\theta\}$ or $\displaystyle\int \underline{\underline{\tan\theta}}\,.\,4\sin 2\theta\ \{\mathrm{d}\theta\}$	$\underline{\underline{\text{M1}}}$
	$= \displaystyle\int 8\sin^2\theta\,\mathrm{d}\theta$	A1
	$3 = 4\sin^2\theta$ **or** $\dfrac{3}{4} = \sin^2\theta$ **or** $\sin\theta = \dfrac{\sqrt{3}}{2} \Rightarrow \theta = \dfrac{\pi}{3}$ $\{x = 0 \to \theta = 0\}$	B1
		(5)
8(c)	$= \{8\}\displaystyle\int\left(\frac{1-\cos 2\theta}{2}\right)\mathrm{d}\theta$ $\left\{= \displaystyle\int(4 - 4\cos 2\theta)\,\mathrm{d}\theta\right\}$	M1
	$= \{8\}\left(\dfrac{1}{2}\theta - \dfrac{1}{4}\sin 2\theta\right)$ $\{= 4\theta - 2\sin 2\theta\}$	M1 A1
	$\left\{\displaystyle\int_0^{\frac{\pi}{3}} 8\sin^2\theta\,\mathrm{d}\theta = 8\left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{3}}\right\} = 8\left(\left(\frac{\pi}{6} - \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)\right) - (0+0)\right)$	
	$= \dfrac{4}{3}\pi - \sqrt{3}$ o.e.	A1
		(4)
		(15 marks)

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Question	Scheme		Marks
9(a)	*Working parametrically:*		
	$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$ or $y = e^{t\ln 2} - 1$		
	$\{x = 0 \Rightarrow\}\ 0 = 1 - \frac{1}{2}t \Rightarrow t = 2$	Applies $x = 0$ to obtain a value for t.	M1
	When $t = 2$, $y = 2^2 - 1 = 3$	Correct value for y.	A1
			(2)
9(b)	$\{y = 0 \Rightarrow\}\ 0 = 2^t - 1 \Rightarrow t = 0$	Applies $y = 0$ to obtain a value for t. (Must be seen in part (b)).	M1
	When $t = 0$, $x = 1 - \frac{1}{2}(0) = 1$	$x = 1$	A1
			(2)
9(c)	$\frac{dx}{dt} = -\frac{1}{2}$ and either $\frac{dy}{dt} = 2^t \ln 2$ or $\frac{dy}{dt} = e^{t\ln 2}\ln 2$		B1
	$\frac{dy}{dx} = \frac{2^t \ln 2}{-\frac{1}{2}}$	Attempts their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$.	M1
	At A, $t = "2"$, so $m(\mathbf{T}) = -8\ln 2 \Rightarrow m(\mathbf{N}) = \frac{1}{8\ln 2}$	Applies $t = "2"$ and $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1
	$y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or equivalent.		M1 A1 oe cso
			(5)
9(d)	$\text{Area}(R) = \int (2^t - 1).\left(-\frac{1}{2}\right) dt$	Complete substitution for both y and dx	M1
	$x = -1 \rightarrow t = 4$ and $x = 1 \rightarrow t = 0$		B1
	$= \left\{-\frac{1}{2}\right\}\left(\frac{2^t}{\ln 2} - t\right)$ Either $2^t \rightarrow \frac{2^t}{\ln 2}$ or $(2^t - 1) \rightarrow \frac{(2^t)}{\pm\alpha(\ln 2)} - t$ or $(2^t - 1) \rightarrow \pm\alpha(\ln 2)(2^t) - t$		M1*
	$(2^t - 1) \rightarrow \frac{2^t}{\ln 2} - t$		A1
	$\left\{-\frac{1}{2}\left[\frac{2^t}{\ln 2} - t\right]_4^0\right\} = -\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right) - \left(\frac{16}{\ln 2} - 4\right)\right)$		
	Depends on the previous method mark. Substitutes their changed limits in t and subtracts either way round.		dM1*

Question	Scheme		Marks
	$= \frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			(6)
			(16 marks)

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Question	Scheme	Marks
10(a)	$A = \int_0^3 \sqrt{(3-x)(x+1)}\ \mathrm{d}x\ ,\ \ x = 1 + 2\sin\theta$	
	$\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$	B1
	$\left\{\int \sqrt{(3-x)(x+1)}\ \mathrm{d}x \ \text{ or } \int \sqrt{(3+2x-x^2)}\ \mathrm{d}x\right\}$	
	$= \int \sqrt{\big(3-(1+2\sin\theta)\big)\big((1+2\sin\theta)+1\big)}\ \ 2\cos\theta\ \{\mathrm{d}\theta\}$	M1
	$= \int \sqrt{(2-2\sin\theta)(2+2\sin\theta)}\ \ 2\cos\theta\ \{\mathrm{d}\theta\}$	
	$= \int \sqrt{\left(4-4\sin^2\theta\right)}\ \ 2\cos\theta\left\{\mathrm{d}\theta\right\}$	
	$= \int \sqrt{\left(4-4(1-\cos^2\theta\right)}\ \ 2\cos\theta\left\{\mathrm{d}\theta\right\}$ or $\int \sqrt{4\cos^2\theta}\ \ 2\cos\theta\left\{\mathrm{d}\theta\right\}$	M1
	$= 4\int \cos^2\theta\,\mathrm{d}\theta,\ \ \{k = 4\}$	A1
	$0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\dfrac{1}{2} \Rightarrow\ \underline{\theta = -\dfrac{\pi}{6}}$ **and** $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1 \Rightarrow\ \underline{\theta = \dfrac{\pi}{2}}$	B1
		(5)
10(b)	$\left\{k\int \cos^2\theta\left\{\mathrm{d}\theta\right\}\right\} = \{k\}\int\left(\dfrac{1+\cos 2\theta}{2}\right)\left\{\mathrm{d}\theta\right\}$	M1
	$= \{k\}\left(\dfrac{1}{2}\theta + \dfrac{1}{4}\sin 2\theta\right)$	M1
	$\left\{\text{So } 4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\,\mathrm{d}\theta = \left[2\theta + \sin 2\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{2}}\right\}$ $= \left(2\left(\dfrac{\pi}{2}\right) + \sin\left(\dfrac{2\pi}{2}\right)\right) - \left(2\left(-\dfrac{\pi}{6}\right) + \sin\left(-\dfrac{2\pi}{6}\right)\right)$	
	$\left\{= (\pi) - \left(-\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{2}\right)\right\} = \dfrac{4\pi}{3} + \dfrac{\sqrt{3}}{2}$	A1 **cao cso**
		(3)
		(8 marks)

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Question	Scheme	Marks
11(a)	$x = 3\theta\sin\theta,\ \ y = \sec^3\theta,\ \ 0 \le \theta < \frac{\pi}{2}$	
	$\{\text{When } y = 8,\}\ 8 = \sec^3\theta \Rightarrow \cos^3\theta = \frac{1}{8} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ $k \text{ (or } x) = 3\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{3}\right)$	M1
	so $k \text{ (or } x) = \frac{\sqrt{3}\pi}{2}$	A1
		(2)
11(b)	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 3\sin\theta + 3\theta\cos\theta$	B1
	$\left\{\int y\frac{\mathrm{d}x}{\mathrm{d}\theta}\{\mathrm{d}\theta\}\right\} = \int(\sec^3\theta)(3\sin\theta + 3\theta\cos\theta)\{\mathrm{d}\theta\}$	M1
	$= 3\int \theta\sec^2\theta + \tan\theta\sec^2\theta\ \mathrm{d}\theta$	A1 *
	$x = 0$ and $x = k \Rightarrow \underline{\alpha = 0}$ and $\underline{\beta = \frac{\pi}{3}}$	B1
		(4)
11(c)	$\left\{\int \theta\sec^2\theta\,\mathrm{d}\theta\right\} = \theta\tan\theta - \int\tan\theta\{\mathrm{d}\theta\}$	M1 dM1
	$= \theta\tan\theta - \ln(\sec\theta)$ **or** $= \theta\tan\theta + \ln(\cos\theta)$	A1
	$\left\{\int \tan\theta\sec^2\theta\,\mathrm{d}\theta\right\} = \frac{1}{2}\tan^2\theta$ or $\frac{1}{2}\sec^2\theta$ or $\frac{1}{2u^2}$ where $u = \cos\theta$ or $\frac{1}{2}u^2$ where $u = \tan\theta$	M1 A1
	$\{\text{Area}(R)\} = \left[3\theta\tan\theta - 3\ln(\sec\theta) + \frac{3}{2}\tan^2\theta\right]_0^{\frac{\pi}{3}}$ **or** $\left[3\theta\tan\theta - 3\ln(\sec\theta) + \frac{3}{2}\sec^2\theta\right]_0^{\frac{\pi}{3}}$	
	$= \left(3\left(\frac{\pi}{3}\right)\sqrt{3} - 3\ln 2 + \frac{3}{2}(3)\right) - (0)$ **or** $\left(3\left(\frac{\pi}{3}\right)\sqrt{3} - 3\ln 2 + \frac{3}{2}(4)\right) - \left(\frac{3}{2}\right)$	
	$= \frac{9}{2} + \sqrt{3}\pi - 3\ln 2$ **or** $\frac{9}{2} + \sqrt{3}\pi + 3\ln\left(\frac{1}{2}\right)$ **or** $\frac{9}{2} + \sqrt{3}\pi - \ln 8$ **or** $\ln\left(\frac{1}{8}e^{\frac{9}{2}+\sqrt{3}\pi}\right)$	A1 o.e.
		(6)
		(12 marks)

Question	Scheme		Marks
12(a)	$1 = A(5-P) + BP$	Can be implied.	M1
	$A = \frac{1}{5},\ B = \frac{1}{5}$	Either one.	A1
	giving $\frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}$		A1 **cao, aef**
			(3)
12(b)	$\int \frac{1}{P(5-P)}\,\mathrm{d}P = \int \frac{1}{15}\,\mathrm{d}t$		B1
	$\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t \ (+c)$		M1* A1ft
	$\{t=0,\ P=1 \Rightarrow\}\quad \frac{1}{5}\ln 1 - \frac{1}{5}\ln(4) = 0 + c \quad \left\{\Rightarrow\ c = -\frac{1}{5}\ln 4\right\}$		dM1*
	e.g.: $\frac{1}{5}\ln\left(\frac{P}{5-P}\right) = \frac{1}{15}t - \frac{1}{5}\ln 4$	Using any of the subtraction (or addition) laws for logarithms CORRECTLY	dM1*
	$\ln\left(\frac{4P}{5-P}\right) = \frac{1}{3}t$		
	e.g.: $\frac{4P}{5-P} = \mathrm{e}^{\frac{1}{3}t}$ or e.g.: $\frac{5-P}{4P} = \mathrm{e}^{-\frac{1}{3}t}$	Eliminate ln's correctly.	dM1*
	gives $4P = 5\mathrm{e}^{\frac{1}{3}t} - P\mathrm{e}^{\frac{1}{3}t} \Rightarrow P(4+\mathrm{e}^{\frac{1}{3}t}) = 5\mathrm{e}^{\frac{1}{3}t}$		
	$P = \frac{5\mathrm{e}^{\frac{1}{3}t}}{(4+\mathrm{e}^{\frac{1}{3}t})}\quad \left\{\frac{(\div\ \mathrm{e}^{\frac{1}{3}t})}{(\div\ \mathrm{e}^{\frac{1}{3}t})}\right\}$	Make P the subject.	dM1*
	$P = \frac{5}{(1+4\mathrm{e}^{-\frac{1}{3}t})}$ or $P = \frac{25}{(5+20\mathrm{e}^{-\frac{1}{3}t})}$ etc.		A1
			(8)
12(c)	$1 + 4\mathrm{e}^{-\frac{1}{3}t} > 1 \Rightarrow P < 5$. So population cannot exceed 5000.		B1
			(1)
			(12 marks)

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	Source paper	Question number	New spec references	Question description	New AOs
1	C4 2012	1	2.10, 8.6	Partial fractions, Integration	1.1b
2	C4 Jan 2011	1	8.4, 5.3	Integration	1.1b
3	C4 Jan 2013	2	8.2, 8.5	Integration	1.1b
4	C4 Jan 2012	2	8.2, 8.5	Integration	1.1b,
5	C4 2015	3	6.3, 8.5	Integration	1.1b, 2.1, 3.1a
6	C4 2013	3	8.2, 8.5, 6.4	Integration	1.1b, 2.1
7	C4 Jan 2011	3	2.10, 8.6, 8.7	Algebra and functions, Integration	1.1b, 3.1a
8	C4 2016	6	8.5, 8.6	Integration	1.1b, 2.1, 3.1a
9	C4 Jan 2013	5	3.3, 7.3, 7.5, 8.3, 8.5	Parametric curves and equations, Parametric differentiation, Integration	1.1b, 3.1a
10	C4 2015	6	8.2, 8.3, 8.5	Integration	1.1b, 2.1
11	C4 2017	8	5.4, 7.4, 8.3, 8.5	Parametric curves and equations, Integration	1.1b, 2.1, 3.1a
12	C4 Jan 2012	8	2.10, 8.6, 8.7, 8.8, 2.11	Partial fractions, Integration	1.1b, 2.1, 3.1a, 3.4