

Advanced Subsidiary**Paper 1: Pure Mathematics****PAPER B Mark Scheme**

1	Any reasonable explanation. For example, the student did not correctly find all values of $2x$ which satisfy $\cos 2x = -\frac{\sqrt{3}}{2}$. Student should have subtracted 150° from 360° first, and then divided by 2. N.B. If insufficient detail is given but location of error is correct then mark can be awarded from working in part (b).	B1
		(1 mark)
	$x = 75^\circ$	B1
	$x = 105^\circ$	B1
		(2 marks)
		Total 3 marks

NOTE: 1a: Award the mark for a different explanation that is mathematically correct, provided that the explanation is clear and not ambiguous.

2	Makes an attempt to use Pythagoras' theorem to find $ \mathbf{a} $. For example, $\sqrt{(4)^2 + (-7)^2}$ seen.	M1
	$\sqrt{65}$	A1
	Displays the correct final answer. $\frac{1}{\sqrt{65}}(4\mathbf{i} - 7\mathbf{j})$	A1
		(3 marks)

3	Attempt to multiply the numerator and denominator by $k(8 + \sqrt{3})$. For example, $\frac{6\sqrt{3} - 4}{8 - \sqrt{3}} \times \frac{8 + \sqrt{3}}{8 + \sqrt{3}}$	M1
	Attempt to multiply out the numerator (at least 3 terms correct). $48\sqrt{3} + 18 - 32 - 4\sqrt{3}$	M1
	Attempt to multiply out the denominator (for example, 3 terms correct but must be rational or $64 - 3$ seen or implied). $64 + 8\sqrt{3} - 8\sqrt{3} - 3$	M1
	p and q stated or implied (condone if all over 61). $\frac{44}{61}\sqrt{3} - \frac{14}{61} \text{ or } p = \frac{44}{61}, q = \frac{14}{61}$	A1
		(4 marks)

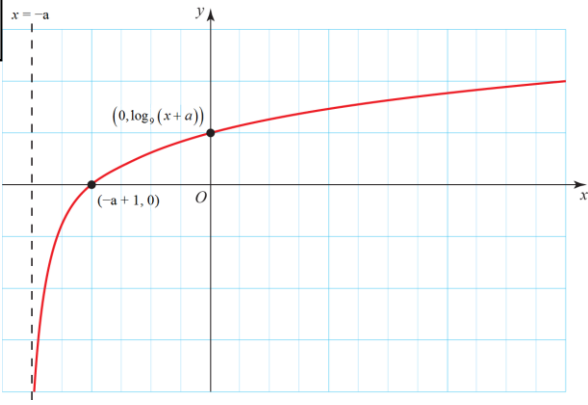
4a	Makes an attempt to expand the binomial expression $(1+x)^3$ (must be terms in x^0, x^1, x^2, x^3 and at least 2 correct).	M1
	$1+3x^2+x^3 < 1+3x+3x^2+x^3$	A1
	$0 < 3x$	A1
	$x > 0^*$ as required.	A1*
		(4 marks)
4b	Picks a number less than or equal to zero, e.g. $x = -1$, and attempts a substitution into both sides. For example, $1+3(-1)^2+(-1)^3 < 1+3(-1)+3(-1)^2+(-1)^3$	M1
	Correctly deduces for their choice of x that the inequality does not hold. For example, $3 \nless 0$	A1
		(2 marks)
		Total 6 marks

5	Uses laws of indices correctly at least once anywhere in solution (e.g. $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$ or $\sqrt{x} = x^{\frac{1}{2}}$ or $x\sqrt{x} = x^{\frac{3}{2}}$ seen or implied).	B1
	Makes an attempt at integrating $h'(x) = 15x^{\frac{3}{2}} - 40x^{-\frac{1}{2}}$ Raising at least one x power by 1 would constitute an attempt.	M1
	Fully correct integration. $6x^{\frac{5}{2}} - 80x^{\frac{1}{2}}$ (no need for $+C$ here).	A1
	Makes an attempt to substitute (4, 19) into the integrated expression. For example, $19 = 6 \times 4^{\frac{5}{2}} - 80 \times 4^{\frac{1}{2}} + C$ is seen.	M1
	Finds the correct value of C. $C = -13$	A1
	States fully correct final answer $h(x) = 6x^{\frac{5}{2}} - 80\sqrt{x} - 13$ or any equivalent form.	A1
		(6 marks)

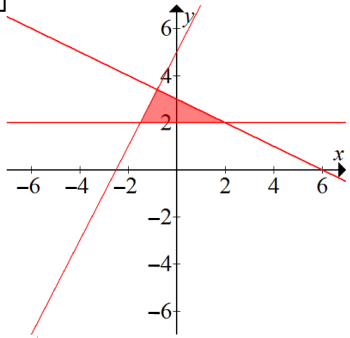
NOTES: Award all 6 marks for a fully correct final answer, even if some working is missing.

6	States $\sin^2 x + \cos^2 x = 1$ or implies this by making a substitution. $8 - 7 \cos x = 6(1 - \cos^2 x)$	M1
	Simplifies the equation to form a quadratic in $\cos x$. $6 \cos^2 x - 7 \cos x + 2 = 0$	M1
	Correctly factorises this equation. $(3 \cos x - 2)(2 \cos x - 1) = 0$ or uses equivalent method for solving quadratic (can be implied by correct solutions).	M1
	Correct solution. $\cos x = \frac{2}{3}$ or $\frac{1}{2}$	A1
	Finds one correct solution for x. ($48.2^\circ, 60^\circ, 311.8^\circ$ or 300°).	A1
	Finds all other solutions to the equation.	A1
		(6 marks)

7a	States or implies the expansion of a binomial expression to the 8th power, up to and including the x^3 term. $(a + b)^8 = {}^8C_0a^8 + {}^8C_1a^7b + {}^8C_2a^6b^2 + {}^8C_3a^5b^3 + \dots$ or $(a + b)^8 = a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + \dots$	M1
	Correctly substitutes 1 and $3x$ into the formula: $(1 + 3x)^8 = 1^8 + 8 \times 1^7 \times 3x + 28 \times 1^6 \times (3x)^2 + 56 \times 1^5 \times (3x)^3 + \dots$	M1
	Makes an attempt to simplify the expression (2 correct coefficients (other than 1) or both $9x^2$ and $27x^3$). $(1 + 3x)^8 = 1^8 + 24x + 28 \times 9x^2 + 56 \times 27x^3 + \dots$	M1 dep
	States a fully correct answer: $(1 + 3x)^8 = 1 + 24x + 252x^2 + 1512x^3 + \dots$	A1
		(4 marks)
7b	States $x = 0.01$ or implies this by attempting the substitution: $1 + 24(0.01) + 252(0.01)^2 + 1512(0.01)^3 + \dots$	M1
	Attempts to simplify this expression (2 calculated terms correct): $1 + 0.24 + 0.0252 + 0.001512$	M1
	$1.266712 = 1.2667 \text{ (5 s.f.)}$	A1
		(3 marks)
		Total 7 marks

8a		Attempt to find intersection with x -axis. For example, $\log_9(x+a)=0$	M1
		Solving $\log_9(x+a)=0$ to find $x=-a+1$, so coordinates of x -intercept are $(-a+1, 0)$ oe	A1
		Substituting $x=0$ to derive $y=\log_9(x+a)$, so coordinates of y -intercept are $(0, \log_9(x+a))$	B1
		Asymptote shown at $x=-a$ stated or shown on graph.	B1
		Increasing log graph shown with asymptotic behaviour and single x -intercept.	M1
		Fully correct graph with correct asymptote, all points labelled and correct shape.	A1
			(6 marks)
8b	$\log_9(x+a)^2 = 2\log_9(x+a)$ seen.	M1	
The graph of $y = \log_9(x+a)^2$ is a stretch, parallel to the y -axis, scale factor 2, of the graph of $y = \log_9(x+a)$.		A1	
			(2 marks)
			Total 8 marks

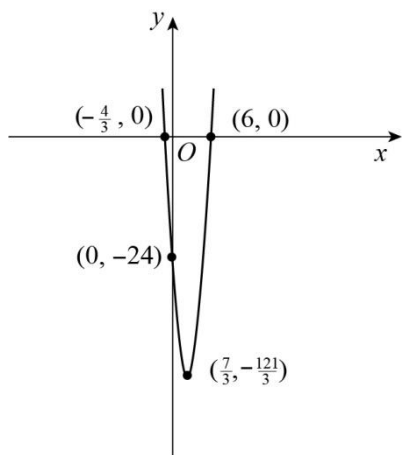
NOTES: 8a: Award all 5 points for a fully correct graph with asymptote and all points labelled, even if all working is not present

9a		Graph of $y = 2x + 5$ drawn.	B1
		Graph of $2y + x = 6$ drawn.	B1
		Graph of $y = 2$ drawn onto the coordinate grid and the triangle correctly shaded.	B1
			(3 marks)
9b	Attempt to solve $y = 2x + 5$ and $2y + x = 6$ simultaneously for y .		M1
	$y = 3.4$		A1
	Base of triangle = 3.5		B1
	Area of triangle = $\frac{1}{2} \times ("3.4" - 2) \times 3.5$		M1
	Area of triangle is 2.45 (units ²).		A1
			(5 marks)
			Total 8 marks

NOTES: 9b: It is possible to find the area of triangle by realising that the two diagonal lines are perpendicular and therefore finding the length of each line using Pythagoras' theorem. Award full marks for a correct final answer using this method.

In this case award the second and third accuracy marks for finding the lengths $\sqrt{2.45}$ and $\sqrt{9.8}$

10a	States that $\tan \theta = \pm \frac{2}{3}$ or $\theta = \tan^{-1} \pm \frac{2}{3}$ (if θ shown on diagram sign must be consistent with this).	M1
	Finds -33.7° (must be negative).	A1
		(2 marks)
10b	Makes an attempt to use the formula $\mathbf{F} = m\mathbf{a}$	M1
	Finds $p = 10$ Note: $8 + p = 6 \times 3 \Rightarrow p = 10$	A1
	Finds $q = -2$ Note: $-10 + q = 6 \times -2 \Rightarrow q = -2$	A1
		(3 marks)
10c	Attempt to find $\mathbf{R}$ (either $6(3\mathbf{i} - 2\mathbf{j})$ or $8\mathbf{i} - 10\mathbf{j} + '10'\mathbf{i} + '-2'\mathbf{j}$).	M1
	Makes an attempt to find the magnitude of their resultant force. For example, $ R = \sqrt{'18'^2 + '12'^2} (= \sqrt{468})$	M1
	Presents a fully simplified exact final answer. $ R = 6\sqrt{13}$	A1
		(3 marks)
		Total 8 marks

11	Attempts to differentiate.	M1
	$f'(x) = 3x^2 - 14x - 24$	A1
	States or implies that the graph of the gradient function will cut the x -axis when $f'(x) = 0$ $f'(x) = 0 \Rightarrow 3x^2 - 14x - 24 = 0$	M1
	Factorises $f'(x)$ to obtain $(3x + 4)(x - 6) = 0$ $x = -\frac{4}{3}, x = 6$	A1
	States or implies that the graph of the gradient function will cut the y -axis at $f'(0)$. Substitutes $x = 0$ into $f'(x)$ Gradient function will cut the y -axis at $(0, -24)$.	M1
	Attempts to find the turning point of $f'(x)$ by differentiating (i.e. finding $f''(x)$)	M1
	$f''(x) = 0 \Rightarrow 6x - 14 = 0 \Rightarrow x = \frac{7}{3}$	A1
	Substitutes $x = \frac{7}{3}$ into $f'(x)$ to obtain $y = -\frac{121}{3}$	A1ft
		A parabola with correct orientation with required points correctly labelled. A1ft
		(9 marks)

NOTES: A mistake in the earlier part of the question should not count against the students for the last part. If a student sketches a parabola with the correct orientation correctly labelled for their values, award the final mark.

Note that a fully correct sketch without all the working but with all points clearly labelled implies 8 marks in this question.

12a	Equates the curve and the line. $x^2 - 8x + 20 = x + 6$	M1
	Simplifies and factorises. $(x - 7)(x - 2) = 0$ (or uses other valid method for solving a quadratic equation).	M1
	Finds the correct coordinates of A. A(2, 8).	A1
	Finds the correct coordinates of B. B(7, 13).	A1
		(4 marks)
12b	Makes an attempt to find the area of the trapezium bounded by $x = 2$, $x = 7$, the x-axis and the line. For example, $\frac{5}{2}(8+13)$ or $\int_2^7 (x+6)dx$ seen.	M1
	Correct answer. Area = 52.5 o.e.	A1
		(2 marks)
12c	$\int_2^7 (x^2 - 8x + 20)dx$.	B1
	Makes an attempt to find the integral. Raising at least one x power by 1 would constitute an attempt.	M1
	Correctly finds $\left[\frac{1}{3}x^3 - 4x^2 + 20x \right]_2^7$	A1
	Makes an attempt to substitute limits into the definite integral. $\left[\left(\frac{343}{3} - 196 + 140 \right) - \left(\frac{8}{3} - 16 + 40 \right) \right]$	M1
	Correct answer seen. $\frac{95}{3}$ or $31.\dot{6}$ oe seen.	A1
		(5 marks)
12d	Understands the need to subtract the two areas. $\pm(52.5 - 31.\dot{6})$	M1
	20.8 units ² seen (must be positive).	A1
		(2 marks)
		Total 13 marks

NOTES: 12a: If A0A0, award A1 for full solution of quadratic equation (i.e. $x = 2$, $x = 7$).

13a	Student completes the square twice. Condone sign errors. $(x-4)^2 - 16 + (y+5)^2 - 25 + 1 = 0$ $(x-4)^2 + (y+5)^2 = 40$	M1
	So centre is $(4, -5)$	A1
	and radius is $\sqrt{40}$	A1
		(3 marks)
13b	Substitutes $x = 10$ into equation (in either form). $10^2 - 8 \times 10 + y^2 + 10y + 1 = 0 \text{ or } (10-4)^2 + (y+5)^2 = 40$	M1
	Rearranges to 3 term quadratic in y $y^2 + 10y + 21 = 0$ (could be in completed square form $(y+5)^2 = 4$)	M1
	Obtains solutions $y = -3, y = -7$ (must give both).	A1
	Rejects $y = -7$ giving suitable reason (e.g. $-7 < -5$) or 'it would be below the centre' or 'AQ must slope upwards' o.e.	B1
		(4 marks)
13c	$m_{AQ} = \frac{-3 - (-5)}{10 - 4} = \frac{1}{3}$	B1
	$m_{l_2} = -3$ (i.e. -1 over their m_{AQ})	B1ft
	Substitutes their Q into a correct equation of a line. For example, $-3 = (-3)(10) + b \text{ or } y + 3 = -3(x - 10)$	M1
	$y = -3x + 27$	A1
		(4 marks)

13d	$\vec{AQ} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ o.e. (could just be in coordinate form).	M1
	$\vec{AP} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$ o.e. so student concludes that point P has coordinates (2, 1).	M1
	Substitutes their P and their gradient $\frac{1}{3}$ (m_{AQ} from 5c) into a correct equation of a line. For example, $1 = \left(\frac{1}{3}\right)(2) + b$ or $y - 1 = \left(\frac{1}{3}\right)(x - 2)$	M1
	$y = \frac{1}{3}x + \frac{1}{3}$	A1
		(4 marks)
13e	$PA = \sqrt{40}$	B1
	Uses Pythagoras' theorem to find $EP = \sqrt{\frac{40}{9}}$.	B1
	Area of $EPA = \frac{1}{2} \times \sqrt{40} \times \sqrt{\frac{40}{9}}$ (could be in two parts).	M1
	Area = $\frac{20}{3}$	A1
		(4 marks)
		Total 19 marks