A level Mathematics Practice Paper –Statistical distributions – Mark scheme

Question	Scheme	Marks
1(a)	awrt $\pm$ 1.40	B1
	$\frac{23-\mu}{5} = "1.40"$ (o.e)	M1A1ft
	$\underline{\mu = 16}$ (or awrt 16.0)	A1
		(4)
1(b)	$\underline{0.4192}$	B1
		(1)
		(5 marks)
2(a)	$(z = \pm)\frac{15-16.12}{1.6}(= -0.70)$	M1
	P(Z < -0.70) = 1 - 0.7580	M1
	= $\underline{0.2420}$ **(awrt 0.242)**	A1
		(3)
2(b)	[P($T < t$)=0.30 implies] $z = \frac{t-16.12}{1.6} = -0.5244$	M1 A1
	$\frac{t-16.12}{1.6} = -0.5244 \Rightarrow t = 16.12 - 1.6 \times "0.5244"$	M1
	t = awrt $\underline{15.28}$ (allow awrt 15.28/9)	A1
		(4)
		(7 marks)

Question	Scheme	Marks
3(a)	$\left[\mathrm{P}\left(M<145\right)=\right]\ \mathrm{P}\left(Z<\dfrac{145-150}{10}\right)$	M1
	$=\mathrm{P}(Z<-0.5)$ or $\mathrm{P}(Z>0.5)$	A1
	= **awrt <u>0.309</u>**	A1
		(3)
3(b)	$\left[\mathrm{P}(B>115)=0.15\Rightarrow\right]\ \dfrac{115-100}{d}=1.0364$ (Calc gives 1.036433...)	M1B1A1
	<u>d = 14.5</u> (Calc gives 14.4727...)	A1
		(4)
3(c)	$[\mathrm{P}(X>\mu+15\mid X>\mu-15)=]\ \dfrac{\mathrm{P}\left(X>\mu+15\right)}{\mathrm{P}\left(X>\mu-15\right)}$	M1
	$=\dfrac{0.35}{1-0.35}$	A1
	$=\underline{\dfrac{7}{13}}$ or **<u>awrt 0.538</u>**	A1
		(3)
		(10 marks)

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Question	Scheme	Marks
4(a)	$\left[\frac{kx^{n+1}}{n+1}\right]_0^1 = 1$	M1 A1
	$k = n + 1$	A1
		(3)
4(b)	$\int_0^1 kx^{n+1}\mathrm{d}x = \left[\frac{kx^{n+2}}{n+2}\right]_0^1$	M1 A1
	$= \frac{n+1}{n+2}$	A1cao
		(3)
4(c)	$\int_0^1 kx^{n+2}\mathrm{d}x = \left[\frac{kx^{n+3}}{n+3}\right]$	M1
	$= \frac{n+1}{n+3}$	A1cao
		(2)
4(d)	Var $(X) = \frac{3}{5} - \left(\frac{3}{4}\right)^2 = \frac{3}{80}$	M1
	Var $(3X) = 9$ Var (X)	M1
	$= \frac{27}{80}$ oe or 0.3375 or 0.338	A1cso
		(3)
		(11 marks)

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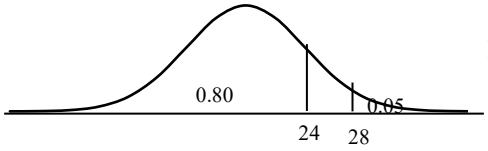
Question	Scheme	Marks
5(a)	 24 and 28 (above the mean) For 0.80 and 0.05 (clearly indicated)	B1 B1
		(2)
5(b)	15%	B1
		(1)
5(c)	$\frac{(28-\mu)}{\sigma}=1.64(49)$ or $\frac{(24-\mu)}{\sigma}=0.84(16)$	M1
	0.8416 and 1.6449 seen	B1
	$\mu=28-1.64(49)\sigma$, $\mu=24-0.84(16)\sigma$	A1 A1
	$24-0.8416\sigma=28-1.6449\sigma$ eliminating μ or σ	M1
	$\sigma=4.9794597...$ **awrt** 4.98	A1
	$\mu=19.809286...$ **awrt** 19.8	A1
		(7)
5(d)	$z=\frac{(12-'19.8...')}{'4.97...'}$	M1
	$P(Z<-1.57)=1-P(Z<1.57)$	dM1
	$1-0.9418=0.0582$ **awrt** 0.06	A1
		(3)
		(13 marks)

Question	Scheme	Marks
6(a)	[Let X be the amount of beans in a tin. $P(X < 200) = 0.1$]	
	$\frac{200-\mu}{7.8} = -1.2816$ [calc gives 1.28155156…]	M1 B1
	μ = 209.996…. awrt 210	A1
		(3)
6(b)	$P(X > 225) = P\left(Z > \frac{225-"210"}{7.8}\right)$	M1
	= $P(Z > 1.92)$ or $1 - P(Z < 1.92)$ (allow 1.93)	A1
	= 1 – 0.9726 = 0.0274 (or better) [calc gives 0.0272037…]	
	= 0.0274	
	= awrt **2.7%** allow **0.027**	A1
		(3)
6(c)	[Let Y be the new amount of beans in a tin]	
	$\frac{210-205}{\sigma} = 2.3263$ or $\frac{200-205}{\sigma} = -2.3263$ [calc gives 2.3263478…]	M1 B1
	$\sigma = \frac{5}{2.3263}$	dM1
	$\sigma = 2.15$ (2.14933…)	A1
		(4)
		(10 marks)

Question	Scheme	Marks
7(a)	The random variable $H \sim$ height of females $\mathrm{P}(H > 170) = \mathrm{P}\left(Z > \frac{170-160}{8}\right)$ $[= \mathrm{P}(Z > 1.25)]$	M1
	$= 1 - 0.8944$	M1
	$= 0.1056$ (calc 0.1056498…) **awrt 0.106** (accept 10.6%)	A1
		(3)
7(b)	$\mathrm{P}(H > 180) = \mathrm{P}\left(Z > \frac{180-160}{8}\right)$ $[= 1 - 0.9938]$	M1
	$= 0.0062$ (calc 0.006209…) awrt 0.0062 or $\frac{31}{5000}$	A1
	$[\mathrm{P}(H > 180 \mid H > 170)] = \frac{0.0062}{0.1056}$	M1
	$= 0.0587$ (calc 0.0587760…) **awrt 0.0587 or 0.0588**	A1
		(4)
7(c)	$\mathrm{P}(H > h \mid H > 170)$ $(= 0.5)$ <u>or</u> $\frac{\mathrm{P}(H > h)}{\mathrm{P}(H > 170)} (= 0.5)$	M1
	$[\mathrm{P}(H > h)] = 0.5 \times "0.1056" = 0.0528$ (calc 0.0528249…) <u>or</u> $[\mathrm{P}(H < h)] = 0.9472$	A1ft
	$\frac{h-160}{8} = 1.62$ (calc 1.6180592…)	M1 B1
	$h =$ awrt 173 cm **awrt 173**	A1
		(5)
		(12 marks)

Question	Scheme	Marks
8(a)	$P(W < 224) = P\left(z < \frac{224-232}{5}\right)$	M1
	$= P(z < -1.6)$	
	$= 1 - 0.9452$	M1
	$= 0.0548$ awrt 0.0548	A1
		(3)
8(b)	$0.5 - 0.2 = 0.3$ 0.3 or 0.7 seen	M1
	$\frac{w-232}{5} = 0.5244$ 0.5244 seen; any z	B1 M1
	$w = 234.622$ awrt 235	A1
		(4)
8(c)	$0.2 \times (1 - 0.2)$	M1
	$2 \times 0.8 \times (1 - 0.8) = 0.32$	M1 A1
		(3)
		(10 marks)

Question	Scheme		Marks
9	N $(0.2n, 0.16n)$	Mean = $0.2n$ and Var = $0.16n$ oe This may be awarded if they appear in the standardisation as $0.2n$ and either $0.16n$ or $\sqrt{0.16n}$	B1
	$P\left(Z > \dfrac{55.5 - 0.2n}{\sqrt{0.16n}}\right) = 0.0401$	Using a continuity correction either 55.5 or 54.5	M1
	$\dfrac{55.5 - 0.2n}{\sqrt{0.16n}} = 1.75$	B1: Using a z = awrt ± 1.75 M1: Standardising using either 55.5, 54.5 or 55 and equal to a z value. Follow through their mean and variance. If they have not given the mean and Var earlier then they must be correct. A1: A correct equation. May be awarded for $\dfrac{55.5 - 0.2n}{\sqrt{0.16n}} = 1.75$ Condone use of inequality sign rather than equals sign.	B1M1A1
	$0.2n + 0.7\sqrt{n} - 55 = 0$	This is dependent on the previous method mark being awarded. Using either the quadratic formula or completing the square or factorising or any correct method to solve **their 3 equations**. If they write the formula written down then it must be correct for their equation. May be implied by correct answer of $\sqrt{n} = 15$ or 342.25 **NB** you may award this mark if they use 54.5 for awrt 14.9, – 18.4, 221 or 337 55 for awrt – 18.4, 223, or – 117 If the answer is not one of these then the method for solving their 3 term equation must be seen.	
	$\sqrt{n} = 15$	Allow 15 or – 18.5 do not need to see n or $\sqrt{n}$ Condone $n = 15$ or $n = -18.5$	A1
	$n = 225$	Cao 225 do not need to see see n or $\sqrt{n}$	A1
			(8 marks)
	Alternative method for last 3 marks $(0.2n - 55)^2 = (-0.7\sqrt{n})^2$ $0.04n^2 - 22.69n + 3080.25 = 0$ $n = 225$ or 1369/4 $n = 225$	M1 solving 3 term quadratic in n as above A1 either 225 or 1369/4 or 342.25 A1 must select 225	

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Question	Scheme	Marks
10(a)	$P(M < 10) = P\left(Z < \frac{12-14}{\sigma}\right) = 0.1$	
	$\Rightarrow \frac{12-14}{\sigma} =, -1.2816$	M1 B1
	$\sigma = 1.5605$..... =awrt 1.56 minutes	A1
		(3)
10(b)	T represents number less than 12 minutes. $T \sim B(15, 0.1)$	B1
	$P(T \leq 1)$	M1
	= 0.549	A1
		(3)
10(c)	[T ~ number of people who take less than 12 mins to complete the test] $T \sim B(n, 0.1)$	
	T can be approximated by N($0.1n$, $0.09n$)	B1
	$P\left(Z < \frac{8.5-0.1n}{\sqrt{0.09n}}\right) = 0.3085$	M1
	$\frac{8.5-0.1n}{\sqrt{0.09n}} = -0.5$ or $\frac{8.5-0.1x^2}{0.3x} = -0.5$	B1 M1 A1
	$0.1n - 0.15\sqrt{n} - 8.5 = 0$ $\sqrt{n} = 10$	M1A1
	$n = 100$	A1cso
		(8)
		(14 marks)

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	Source paper	Question number	New spec references	Question description	New AOs
1	S1 2011	2	A 4.2	Normal distribution	1.1b, 3.1a
2	S1 2011	4	A 4.2	Normal distribution	1.1b, 3.3, 3.4
3	S1 2013R	4	A 3.2, 4.2	Normal distribution, Conditional probability	1.1b, 3.1a, 3.1b, 3.4
4	S1 2015	6	A 3.2, 4.2	Normal distribution	1.1b, 2.1, 3.1a, 3.4
5	S1 2014R	6	A 4.2	Normal distribution	1.1b, 3.1b, 3.4
6	S1 2013	6	A 4.3	Normal distribution	1.1b, 2.1, 3.1b, 3.4
7	S1 2014	7	A 3.2, 4.2	Normal distribution	1.1b, 2.1, 3.1b, 3.3, 3.4
8	S1 Jan 2012	7	A 4.2	Normal distribution, Probability	1.1b, 3.1b, 3.3, 3.4
9	S2 2016	5	A 4.2	Binomial distribution, Normal distribution	1.1b, 1.2, 3.1b, 3.3, 3.4
10	S2 2017	5	A 4.1, 4.2	Binomial distribution, Normal distribution	1.1b, 3.1b, 3.3, 3.4