Year 2 Pure Chapter 3 – Sequences and Series

Arithmetic Sequences

a = 1st term

d = common difference

U_n = nth term

U_1	U_2	U_3	U_n
a	a + d	a + 2d	a + (n − 1)d

Watch Out: Terms and Years

2000	2001	2002	2003
n = 1	n = 2	n = 3	n = 4

Sum of Arithmetic Series

Sequence 5, 7, 9, 11

Series 5 + 7 + 9 + 11

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

or

$$S_n = \frac{n}{2}[a + l]$$

l = last term

Common Ratio Geometric

U_1	U_2	U_3
2	x	18

$$\frac{x}{2} \qquad \frac{18}{x}$$

common ratio $\frac{x}{2} = \frac{18}{x}$

$$x = 6$$

Geometric Sequences

a = 1st term

r = common ratio

U_n = nth term

U_1	U_2	U_3	U_n
a	ar	ar^2	$ar^{(n-1)}$

Sum of Geometric Series

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

or

$$S_n = \frac{a(r^n - 1)}{1 - r}$$

Sum To Infinity (Geometric)

$$S_n = \frac{a}{1 - r}$$

Sigma Notation

$\sum$ *Sum of*

$\sum_{1}^{20}$ *Sum of 1st 20 terms*

Expression to calculate each term.

$\sum 3r+1$

Prove Arithmetic Sum

$S_n = a + (a+d) + (a+2d) + \cdots + (a+(n-1)d)$

Reverse

$S_n = (a+(n-1)d) + \cdots + (a+2d) + (a+d) + a$

Adding:

$2S_n = (2a+(n-1)d) + \cdots + (2a+(n-1)d)$

$$2S_n = n(2a+(n-1)d)$$

$$S_n = \frac{n}{2}(2a+(n-1)d)$$

Prove Geometric Sum

$S_n = a + ar + ar^2 + \cdots + ar^{n-2} + ar^{n-1}$

Multiplying by r

$rS_n = ar + ar^2 + \cdots + ar^{n-2} + ar^{n-1} + ar^n$

Subtracting

$$S_n - rS_n = a - ar^n$$

$$S_n(1-r) = a(1-r^n)$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

Recurrence Relations

The rules require the previous term to calculate the next.

$$U_{n+1} = U_n + 4$$
$$U_1 = 7$$
$$U_2 = 7 + 4 = 11$$
$$U_2 = 11 + 4 = 15$$

Period Sequences

Repeating in a cycle

2, 3, 4, 2, 3, 5, 2, 3, 5.....

Periodic Order = 3

Repeats every 3 terms