

A level Mathematics Practice Paper – Trigonometry (part 3) – Mark scheme

Question	Scheme		Marks
1(a)	$2 \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} = \frac{5}{\sin x}$		B1
	Uses common denominator to give $2 \sin^2 x - \cos^2 x = 5 \cos x$		M1
	Replaces $\sin^2 x$ by $(1 - \cos^2 x)$ to give $2(1 - \cos^2 x) - \cos^2 x = 5 \cos x$		M1
	Obtains $3 \cos^2 x + 5 \cos x - 2 = 0$ ($a=3, b=5, c=-2$)		A1
			(4)
1(b)	Solves $3 \cos^2 x + 5 \cos x - 2 = 0$ to give $\cos x =$		M1
	$\cos x = \frac{1}{3}$ only (rejects $\cos x = -2$)		A1
	So $x = 1.23$ or 5.05		dM1A1
			(4)
1(c)	Either	Or	
	$\tan \theta + \cot \theta \equiv \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$	$\tan \theta + \cot \theta \equiv \tan \theta + \frac{1}{\tan \theta}$	B1
	$\equiv \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$	$\equiv \frac{\tan^2 \theta + 1}{\tan \theta}$	M1
	$\equiv \frac{2}{\sin 2\theta}$	$\equiv \frac{1}{\cos^2 \theta \times \frac{\sin \theta}{\cos \theta}}$ $\equiv \frac{2}{\sin 2\theta}$	M1
	$\equiv 2 \operatorname{cosec} 2\theta$ (so $\lambda = 2$)	$\equiv 2 \operatorname{cosec} 2\theta$ (so $\lambda = 2$)	A1
			(4)
			(12 marks)

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Question	Scheme	Marks
2(a)	$4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta = \frac{4}{\sin^2 2\theta} - \frac{1}{\sin^2 \theta}$	
	$= \frac{4}{(2 \sin \theta \cos \theta)^2} - \frac{1}{\sin^2 \theta}$	B1 B1
		(2)
2(b)	$\frac{4}{(2 \sin \theta \cos \theta)^2} - \frac{1}{\sin^2 \theta} = \frac{4}{4 \sin^2 \theta \cos^2 \theta} - \frac{1}{\sin^2 \theta}$	
	$= \frac{1}{\sin^2 \theta \cos^2 \theta} - \frac{\cos^2 \theta}{\sin^2 \theta \cos^2 \theta}$	M1
	Using $1 - \cos^2 \theta = \sin^2 \theta$ $= \frac{\sin^2 \theta}{\sin^2 \theta \cos^2 \theta}$	M1
	$= \frac{1}{\cos^2 \theta} = \sec^2 \theta$	M1 A1*
		(4)
2(c)	$\sec^2 \theta = 4 \Rightarrow \sec \theta = \pm 2 \Rightarrow \cos \theta = \pm \frac{1}{2}$	M1
	$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$	A1 A1
		(3)
		(9 marks)

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Question	Scheme	Marks
3(a)	$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \frac{1 - \cos 2\theta}{\sin 2\theta}$	M1
	$= \frac{2 \sin^2 \theta}{2 \sin \theta \cos \theta}$	M1 A1
	$= \frac{\sin \theta}{\cos \theta} = \tan \theta$	A1*
		(4)
3(b)(i)	$\tan 15^\circ = \frac{1}{\sin 30^\circ} - \frac{\cos 30^\circ}{\sin 30^\circ}$	M1
	$\tan 15^\circ = \frac{1}{\frac{1}{2}} - \frac{\frac{\sqrt{2}}{2}}{\frac{1}{2}} = 2 - \sqrt{3}$ cso	dM1 A1*
		(3)
	Alternative	
	$\tan 15^\circ = \tan (60^\circ - 45^\circ)$ or $\tan (45^\circ - 30^\circ)$	
	$\tan 15^\circ = \frac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45}$ or $\frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$	M1
	$\tan 15^\circ = \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$ or $\frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$	M1
	Rationalises to produce $\tan 15^\circ = 2 - \sqrt{3}$	A1*
3(b)(ii)	$\tan 2x = 1$	M1
	$2x = 45^\circ$	A1
	$2x = 45^\circ + 180^\circ$	M1
	$x = 22.5^\circ, 112.5^\circ, 202.5^\circ, 295.5^\circ$	A1(any two) A1
		(5)
		(12 marks)

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Question	Scheme	Marks
4(a)	$\operatorname{cosec} 2x + \cot 2x = \frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x}$	M1
	$= \frac{1 + \cos 2x}{\sin 2x}$	M1
	$= \frac{1 + 2 \cos^2 x - 1}{2 \sin x \cos x}$	
	$= \frac{2 \cos^2 x}{2 \sin x \cos x}$	M1 A1
	$= \frac{\cos x}{\sin x} = \cot x$	A1*
		(5)
4(b)	$\operatorname{cosec}(4\theta + 10^\circ) + \cot(4\theta + 10^\circ) = \sqrt{3}$	
	$\cot(2\theta \pm ..^\circ) = \sqrt{3}$	M1
	$2\theta \pm ... = 30^\circ \Rightarrow \theta = 12.5^\circ$	dM1 A1
	$2\theta \pm ... = 180 + PV^\circ \Rightarrow \theta = ..^\circ$	dM1
	$\theta = 102.5^\circ$	A1
		(5)
		(10 marks)

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Question	Scheme	Marks
5(a)	$2 \cot 2x + \tan x \equiv \frac{2}{\tan 2x} + \tan x$	B1
	$\equiv \frac{(1 - \tan^2 x)}{\tan x} + \frac{\tan^2 x}{\tan x}$	M1
	$\equiv \frac{1}{\tan x}$	M1
	$\equiv \cot x$	A1*
		(4)
5(b)	$6 \cot 2x + 3 \tan x = \operatorname{cosec}^2 x - 2 \Rightarrow 3 \cot x = \operatorname{cosec}^2 x - 2$	
	$\Rightarrow 3 \cot x = 1 + \cot^2 x - 2$	M1
	$\Rightarrow 0 = \cot^2 x - 3 \cot x - 1$	A1
	$\Rightarrow \cot x = \frac{3 \pm \sqrt{13}}{2}$	M1
	$\Rightarrow \tan x = \frac{2}{3 \pm \sqrt{13}} \Rightarrow x = ..$	M1
	$\Rightarrow x = 0.294, -2.848, -1.277, 1.865$	A2,1,0
		(6)
		(10 marks)

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Question	Scheme	Marks
6(a)	$\sec 2A + \tan 2A = \frac{1}{\cos 2A} + \frac{\sin 2A}{\cos 2A}$	B1
	$= \frac{1 + \sin 2A}{\cos 2A}$	M1
	$= \frac{1 + 2\sin A \cos A}{\cos^2 A - \sin^2 A}$	M1
	$= \frac{\cos^2 A + \sin^2 A + 2\sin A \cos A}{\cos^2 A - \sin^2 A}$	
	$= \frac{(\cos A + \sin A)(\cos A + \sin A)}{(\cos A + \sin A)(\cos A - \sin A)}$	M1
	$= \frac{\cos A + \sin A}{\cos A - \sin A}$	A1*
		(5)
6(b)	$\sec 2\theta + \tan 2\theta = \frac{1}{2} \Rightarrow \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} = \frac{1}{2}$	
	$\Rightarrow 2\cos \theta + 2\sin \theta = \cos \theta - \sin \theta$	
	$\Rightarrow \tan \theta = -\frac{1}{3}$	M1 A1
	$\Rightarrow \theta = \text{awrt } 2.820, 5.961$	dM1A1
		(4)
		(9 marks)

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Question	Scheme	Marks
7(a)	$R = 25$	B1
	$\tan \alpha = \frac{24}{7} \Rightarrow \alpha = (\text{awrt}) 73.7^\circ$	M1 A1
		(3)
7(b)	$\cos(2x + \text{their } \alpha) = \frac{12.5}{\text{their } R}$	M1
	$2x + \text{their } \alpha' = 60^\circ$	A1
	$2x + \text{their } \alpha' = \text{their } 300^\circ \text{ or their } 420^\circ \Rightarrow x = ..$	M1
	$x = \text{awrt } 113.1^\circ, 173.1^\circ$	A1 A1
		(5)
7(c)	Attempts to use $\cos 2x = 2\cos^2 x - 1$ AND $\sin 2x = 2\sin x \cos x$ in the expression	M1
	$14\cos^2 x - 48\sin x \cos x = 7(\cos 2x + 1) - 24\sin 2x$	
	$= 7\cos 2x - 24\sin 2x + 7$	A1
		(2)
7(d)	$14\cos^2 x - 48\sin x \cos x = R\cos(2x + \alpha) + 7$	
	Maximum value = ' R ' + ' c '	M1
	$= 32 \text{ cao}$	A1
		(2)
		(12 marks)

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Question	Scheme	Marks
8(a)	$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$	M1A1
	$\frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{1 - \frac{\sin A \sin B}{\cos A \cos B}} \quad (\div \cos A \cos B)$	M1
	$= \frac{\tan A + \tan B}{1 - \tan A \tan B}$	A1 *
		(4)
8(b)	$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{\tan \theta + \frac{\tan \pi}{6}}{1 - \frac{\tan \theta \tan \pi}{6}}$	M1
	$= \frac{\tan \theta + \frac{1}{\sqrt{3}}}{1 - \tan \theta \frac{1}{\sqrt{3}}}$	M1
	$= \frac{\sqrt{3} \tan \theta + 1}{\sqrt{3} - \tan \theta}$	A1 *
		(3)
8(c)	$\tan\left(\theta + \frac{\pi}{6}\right) = \tan(\pi - \theta)$	M1
	$\left(\theta + \frac{\pi}{6}\right) = (\pi - \theta)$	M1
	$\theta = \frac{5}{12} \pi$	M1 A1
	$\tan\left(\theta + \frac{\pi}{6}\right) = \tan(2\pi - \theta)$	M1
	$\theta = \frac{11}{12} \pi$	A1
		(6)
		(13 marks)

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Question	Scheme	Marks
9(a)	$\sin 2x - \tan x = 2 \sin x \cos x - \tan x$	M1
	$= \frac{2 \sin x \cos^2 x}{\cos x} - \frac{\sin x}{\cos x}$	M1
	$= \frac{\sin x}{\cos x} \times (2 \cos^2 x - 1)$	
	$= \tan x \cos 2x$	dM1 A1*
		(4)
9(b)	$\tan x \cos 2x = 3 \tan x \sin x \Rightarrow \tan x(\cos 2x - 3 \sin x) = 0$	
	$\cos 2x - 3 \sin x = 0$	M1
	$\Rightarrow 1 - 2 \sin^2 x - 3 \sin x = 0$	M1
	$\Rightarrow 2 \sin^2 x + 3 \sin x - 1 = 0 \Rightarrow \sin x = \frac{-3 \pm \sqrt{17}}{4} \Rightarrow x = \dots$	M1
	Two of $x = 16.3^\circ, 163.7^\circ, 0, 180^\circ$	A1
	All four of $x = 16.3^\circ, 163.7^\circ, 0, 180^\circ$	A1
		(5)
		(9 marks)

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	Source paper	Question number	New spec references	Question description	New AOs
1	C3 June 2014R	3	5.4, 5.5, 5.6, 5.7, 5.8	Trigonometry	1.1b, 1.2, 2.1,
2	C3 2012	5	5.4, 5.5, 5.6, 5.7, 5.8	Trigonometry	1.1b, 2.1, 3.1a
3	C3 2011	6	5.3, 5.4, 5.6, 5.7, 5.8	Trigonometry	1.1b, 2.1, 3.1a
4	C3 June 2014	7	5.5, 5.6, 5.7, 5.8	Trig identities and equation solving	1.2, 1.1b, 2.1, 2.2a
5	C3 2016	8	5.5, 5.6, 5.7, 5.8	Proof and equation	1.1b, 2.1, 3.1a
6	C3 2015	8	5.5, 5.6, 5.7, 5.8	Proof using sec and double angle formulae	1.1b, 1.2, 2.1, 3.1a
7	C3 2012	8	5.6, 5.7, 5.8	Trigonometry	1.1b, 2.1, 2.1a,
8	C3 Jan 2012	8	5.3, 5.6, 5.7, 5.8	Trigonometry	1.1b, 2.1, 2.2a, 3.1a
9	C3 2017	9	5.5, 5.6, 5.7	Trig proof, equation involving double angles	1.1b, 1.2, 2.1, 3.1a