Exam Questions – Quadratics (70 mins)

1. Given that the equation $kx^2 + 12x + k = 0$, where k is a positive constant, has equal roots, find the value of k.

(Total 4 marks)

$a = k \quad b = 12 \quad c = k$

$b^2 - 4ac = 0$

$144 - 4(k)(k) = 0$

$144 - 4k^2 = 0$

$4k^2 = 144$

$k^2 = 36 \quad \therefore k = 6$

2. The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots.

Find the value of p.

$a = 1 \quad b = 3p \quad c = p$

$b^2 - 4ac = 0$

$(3p)^2 - 4(1)(p) = 0$

$9p^2 - 4p = 0$

$p(9p - 4) = 0$

$p = 0$ or $4/9$

$\therefore p = 4/9$

(Total 4 marks)

3. The equation $x^2 + 2px + (3p + 4) = 0$, where p is a positive constant, has equal roots.

(a) Find the value of p.

(4)

$a = 1 \quad b = 2p \quad c = 3p + 4$

$b^2 - 4ac = 0$

$(2p)^2 - 4(1)(3p + 4) = 0$

$4p^2 - 12p + 16 = 0$

$p = -1$ or 4

$\therefore p = 4$

(b) For this value of p, solve the equation $x^2 + 2px + (3p + 4) = 0$.

$x^2 + 8x + 16 = 0$

$(x + 4)(x + 4) = 0$

$x = -4$

(2)
(Total 6 marks)

4. (a) Show that $x^2 + 6x + 11$ can be written as

$$(x + p)^2 + q$$

where p and q are integers to be found.

(2)

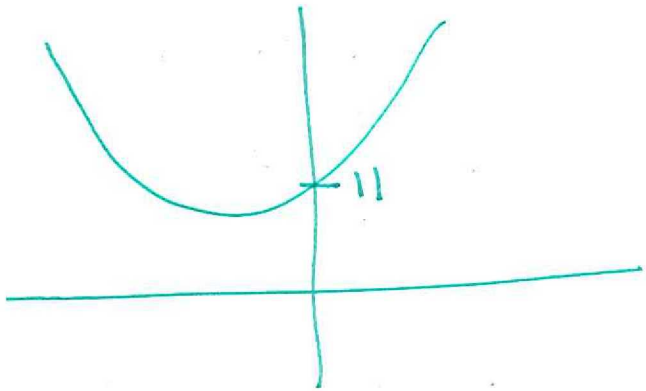
$(x+3)^2 - 9 + 11$

$= (x+3)^2 + 2$

$p = 3 \quad q = 2$

(b) In the space below, sketch the curve with equation $y = x^2 + 6x + 11$, showing clearly any intersections with the coordinate axes.

(2)



(c) Find the value of the discriminant of $x^2 + 6x + 11$

$a = 1 \quad b = 6 \quad c = 11$

$b^2 - 4ac$

$= 6^2 - 4(1)(11)$

$= 36 - 44$

$= -8$

(2)
(Total 6 marks)

5. (a) Solve the equation $4x^2 + 12x = 0$.

(3)

$4x(x+3) = 0$

$x = 0$

or

$x = -3$

$f(x) = 4x^2 + 12x + c$,

where c is a constant.

(b) Given that $f(x) = 0$ has equal roots, find the value of c and hence solve $f(x) = 0$.

$4x^2 + 12x + c = 0$

$b^2 - 4ac = 0$

$144 - (4)(4)(c) = 0$

$144 - 16c = 0$

$c = 9$

hence $4x^2 + 12x + 9 = 0$

$(2x+3)(2x+3) = 0$

$x = -\frac{3}{2}$

(4)
(Total 7 marks)

6. $f(x) = x^2 - kx + 9$, where k is a constant.

(a) Find the set of values of k for which the equation $f(x) = 0$ has equal real roots.

(4)

$a=1 \quad b=-k \quad c=9$

$b^2 - 4ac = 0$

$(-k)^2 - 4(1)(9) = 0$

$k^2 - 36 = 0$

$k = \pm 6$

Given that $k = 4$,

(b) express $f(x)$ in the form $(x - p)^2 + q$, where p and q are constants to be found.

(3)

$x^2 - 4x + 9$

$= (x-2)^2 + 5$

(c) write down the minimum value of $f(x)$ and the value of x for which this occurs.

$(2, 5)$

(2)

(Total 9 marks)

7. Given that

$$f(x) = x^2 - 6x + 18, \quad x \geq 0,$$

(a) express $f(x)$ in the form $(x - a)^2 + b$, where a and b are integers.

(3)

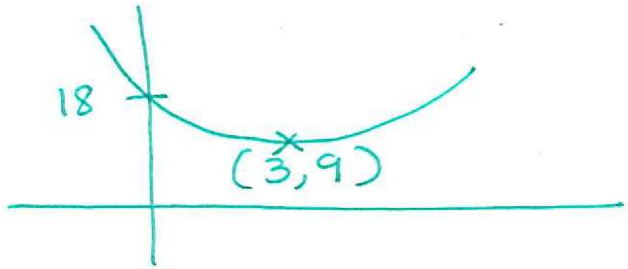
$(x-3)^2 - 9 + 18$

$= (x-3)^2 + 9$

The curve C with equation $y = f(x)$, $x \geq 0$, meets the y-axis at P and has a minimum point at Q.

(b) Sketch the graph of C, showing the coordinates of P and Q.

(4)



The line $y = 41$ meets C at the point R.

(c) Find the x-coordinate of R, giving your answer in the form $p + q\sqrt{2}$, where p and q are integers.

(5)

$x^2 - 6x + 18 = 41$

$x^2 - 6x - 23 = 0$

complete square

$(x-3)^2 - 32 = 0$

$x - 3 = \pm\sqrt{32}$

$x = 3 \pm \sqrt{32}$

$x = 3 \pm 4\sqrt{2}$

(Total 12 marks)

8.

$$x^2 + 2x + 3 \equiv (x + a)^2 + b.$$

(a) Find the values of the constants a and b.

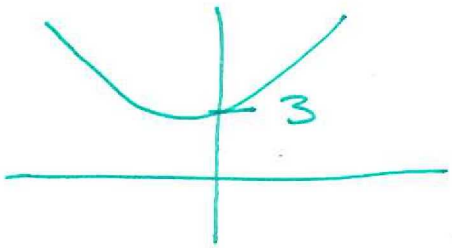
(2)

$(x+1)^2 - 1 + 3$

$(x+1)^2 + 2$

(b) In the space provided below, sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes.

(3)



(c) Find the value of the discriminant of $x^2 + 2x + 3$. Explain how the sign of the discriminant relates to your sketch in part (b).

(2)

$a = 1 \quad b = 2 \quad c = 3$

$b^2 - 4ac$

$= 4 - 4(1)(3)$

$= 4 - 12$

$= -8 \qquad -8 < 0 \therefore$ no real roots

The equation $x^2 + kx + 3 = 0$, where k is a constant, has equal roots.

(d) Find the values of k, giving your answer in surd form.

$a = 1 \quad b = k \quad c = 3$

$(k)^2 - 4(1)(3) = 0$

$k^2 - 12 = 0$

$k^2 = 12$

$k = \pm\sqrt{12}$

(4)

(Total 11 marks)