Pearson Edexcel

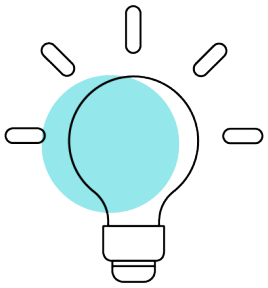
Topic Test Summer 2022

Pearson Edexcel GCE Mathematics (9MA0)

Paper 3 – Statistics

Topic 4: Discrete probability distributions; normal approximation

Contents



Mark Scheme

Question T4_Q1

Qu 1	Scheme	Marks	AO
(a)	c: 0 1 2 3 4 5 6 7 8 P($C = c$): $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$	B1 B1ft (2)	1.2 1.2
(b)	P($C < 4$) = $\frac{4}{9}$ (accept 0.444 or better)	B1 (1)	3.4
(c)	Probability lower than expected suggests model is <u>not</u> good	B1ft (1)	3.5a
(d)	e.g. Cloud cover will vary from month to month and place to place So e.g. use a non-uniform distribution	B1 (1)	3.5c
		(5 marks)	

Notes

(a) 1st B1 for a correct set of values for c. Allow $\{\frac{1}{8}, \frac{2}{8}, ... \frac{8}{8}\}$

2nd B1ft for correct probs from their values for c, consistent with discrete uniform distrib'n
Maybe as a prob. function. Allow P($X = x$) = $\frac{1}{9}$ for $0 \leqslant x \leqslant 8$ provided x = {0, 1, 2, …, 8} is clearly defined somewhere.

(b) B1 for using correct model to get $\frac{4}{9}$ (o.e.)

SC **Sample space {1, …, 8}** If scored B0B1 in (a) for this allow P($C < 4$) = $\frac{3}{8}$ to score B1 in (b)

(c) B1ft for comment that states that the model proposed is or is not a good one based on their model in part (a) and their probability in (b)
|(b) – 0.315| > 0.05 Allow e.g. "it is not suitable"; "it is not accurate" etc
|(b) – 0.315| ⩽ 0.05 Allow a comment that suggests it <u>is</u> suitable
No prob in (b) Allow a comparison that mentions 50% or 0.5 and rejects the model
No prob in (b) and no 50% or 0.5 or **(b) > 1** scores B0
Ignore any comments about location or weather patterns.

(d) B1 for a sensible refinement considering variations in month or location
Just saying "not uniform" is B0
Context & "non-uniform" Allow mention of different locations, months <u>and</u> non-uniform
<u>or</u> use more locations to form a new distribution with probabilities based on frequencies
Context & "binomial" Allow mention of different locations, months <u>and</u> binomial
Just refined model Model must be outlined and discrete and non-uniform
e.g. higher probabilities for more cloud cover <u>or</u> lower probabilities for less cloud cover
Continuous model Any model that is based on a continuous distribution. e.g. normal is B0

Question T4_Q2

Qu 3	Scheme	Marks	AO
(a)	The probability of a dart hitting the target is constant (from child to child and for each throw by each child) (o.e.) The throws of each of the darts are independent (o.e.)	B1 B1 (2)	1.2 1.2
(b)	[P($H \geqslant 4$) = 1 – P($H \leqslant 3$) = 1 – 0.9872 = 0.012795.. =] awrt **0.0128**	B1 (1)	1.1b
(c)	P($F = 5$) = $0.9^4 \times 0.1$, = 0.06561 = awrt **0.0656**	M1, A1 (2)	3.4 1.1b
(d)	<table><tr><td>n</td><td>1</td><td>2</td><td>…</td><td>10</td></tr><tr><td>P($F = n$)</td><td>0.01</td><td>0.01 + α</td><td>…</td><td>0.01+9α</td></tr></table> Sum of probs = 1 $\Rightarrow \frac{10}{2}[2\times0.01+9\alpha]=1$ [i.e. $5(0.02 + 9\alpha) = 1$ or $0.1 + 45\alpha = 1$] so α = **0.02**	M1 M1A1 A1 (4)	3.1b 3.1a 1.1b 1.1b
(e)	P($F = 5$| Thomas' model) = **0.09**	B1ft (1)	3.4
(f)	Peta's model assumes the probability of hitting target is constant (o.e.) **and** Thomas' model assumes this probability increases with each attempt(o.e.)	B1 (1)	3.5a
		(11 marks)	

Notes

(a) 1st B1 for stating that the probability (or possibility or chance) is constant (or fixed or same)
2nd B1 for stating that throws are independent ["trials" are independent is B0]

(b) B1 for awrt 0.0128 (found on calculator)

(c) M1 for a probability expression of the form $(1-p)^4 \times p$ where $0 < p < 1$
A1 for awrt 0.0656
SC Allow M1A0 for answer only of 0.066

(d) 1st M1 for setting up the distribution of F with at least 3 correct values of n and P($F = n$) in terms of α. (Can be implied by 2nd M1 or 1st A1)
2nd M1 for use of sum of probs = 1 **and** clear summation or use of arithmetic series formula (allow 1 error or missing term). (Can be implied by 1st A1)
1st A1 for a correct equation for α
2nd A1 for α = 0.02 (must be exact and come from correct working)

(e) B1ft for value resulting from $0.01 + 4 \times$ "their α" (provided α and the answer are probs)
Beware If their answer is the same as their (c) (or a rounded version of their (c)) score B0

(f) B1 for a suitable comment about the probability of hitting the target
ALT Allow idea that Peta's model suggests the dart may never hit the target but Thomas' says that it will hit at least once (in the first 10 throws).

Question T4_Q3

Question	Scheme	Marks	AOs
4 (a)	$\frac{132}{184} = 0.71739...$ awrt **0.717**	B1	1.1b
		(1)	
(b)(i)	$P(X \geq 6) = 1 - P(X \leq 5)$ or $P([X =]6) + P([X =]7) + P([X =]8)$	M1	3.4
	$= 1 - 0.296722...$ awrt **0.703**	A1	1.1b
		(2)	
(b)(ii)	$184 \times P(X = 7)$ $[= 184 \times 0.2811\ldots]$	M1	1.1b
	$= 51.7385...$ awrt **51.7**	A1	1.1b
		(2)	
(c)	Part (a) and part (b)(i) are similar **and** the expected number of 7s (51.7 or 0.281) matches with the number of 7s found in the data set (52 or 0.283) so Magali's model is supported.	B1ft	3.5a
		(1)	
(d)	$\frac{23}{28} = 0.82142...$ awrt **0.821**	B1	1.1b
		(1)	
(e)	Any one of… • Part (d)/'0.821' differs from part (a)/(b)(i)/(0.7…) • there is a **greater/different probability of high cloud cover/more likely to have high cloud cover** if the previous day had high cloud cover • **independence(o.e.)** does not hold	B1	2.4
	…therefore Magali's (binomial) model may not be suitable.	dB1	3.5a
		(2)	
			(9 marks)

Notes	
	Allow fractions, decimals or percentages throughout this question.
(a)	Allow equivalent fraction, e.g. $\frac{33}{46}$
(b)(i)	**M1:** for writing or using $1 - P(X \leq 5)$ or $P(X = 6) + P(X = 7) + P(X = 8)$ **A1:** awrt 0.703 (correct answer scores 2 out of 2)
(b)(ii)	**M1:** for $184 \times P(X = 7)$ o.e. e.g., $184 \times [P(X \leq 7) - P(X \leq 6)]$ **A1:** awrt 51.7
(c)	**B1ft:** comparing '0.717' with '0.703' and '51.7 or '0.281' with 52 or 0.283 **and** concluding that Magali's model **is supported** (must be comparing prob. with prob. and days with days). Allow not supported or mixed conclusions if consistent with their f.t. answers in (a) and (b)
(e)	**B1:** Any bullet point **dB1:** (dep on previous B1) for Magali's model may not be suitable (o.e.) Condone not accurate for not suitable **SC:** part (d) is similar to part (a)/(b)(i) **and** a compatible conclusion (i.e. Magali's model is supported) to score B1B1.

Question T4_Q4

Qu 1	Scheme	Marks	AO
(a)	**Disadvantage:** e.g. Not random; cannot use (reliably) for inferences	B1 **(1)**	1.1b
(b)	[Sight or correct use of] $X \sim B(36, 0.08)$	M1	3.3
(i)	$P(X = 4) = 0.167387\ldots$ awrt **0.167**	A1	1.1b
(ii)	$[P(X \ldots 7) = 1 - P(X \leq 6) =]$ 0.022233… awrt **0.0222**	A1 **(3)**	1.1b
(c)	P(In dance club and dance tango) = 0.4×0.08 = **0.032** or $\frac{4}{125}$ or **3.2%**	B1 **(1)**	1.1b
(d)	[Let T = those who can dance the Tango. Sight or use of] $T \sim B(50, \text{“}0.032\text{”})$	M1	3.3
	$[P(T < 3) = P(T \leq 2) =]$ 0.7850815… awrt **0.785**	A1 **(2)**	1.1b
		(7 marks)	

Notes

(a) B1 for a suitable disadvantage:

Allow (B1)	Do NOT allow (B0)
Not random or less random (o.e.)	Not representative
Cannot use (reliably) for inferences	Less accurate
(More likely to be) biased	Any comment based on time or cost
	Any mention of skew
	Any mention of non-response

(b) M1 for sight of B(36, 0.08) Allow in words: binomial with $n = 36$ and $p = 0.08$
may be implied by one correct answer to 2sf or sight of $P(X \leq 6) = 0.97776\ldots$ i.e. awrt 0.98
Allow for $36C4 \times 0.08^4 \times 0.92^{32}$ as this is "correct use"

(i) 1st A1 for awrt 0.167 NB An answer of just awrt 0.167 scores M1(⇒)1st A1

(ii) 2nd A1 for awrt 0.0222

(c) B1 for 0.032 o.e. (Can allow for sight of 0.4×0.08)

(d) M1 for sight of B(50, “0.032”) ft their answer to (c) provided it is a probability ≠ 0.08
may be implied by correct answer
or sight of $[P(T \leq 3)] = 0.924348\ldots$ i.e. awrt 0.924 or $P(T \leq 2)$ as part of $1 - P(T \leq 2)$ calc.
A1 for awrt 0.785

MR Allow MR of 50 (e.g. 30) provided clearly attempting $P(T \leq 2)$ and score M1A0

Question T4_Q5

Qu 6	Scheme	Marks	AO
(a)	[Sum of probs = 1 implies] $\log_{36} a + \log_{36} b + \log_{36} c = 1$	M1	3.1a
	$\Rightarrow \log_{36}(abc) = 1$ so $abc = 36$	A1	3.4
	All probabilities greater than 0 implies each of a, b and $c > 1$	B1	2.2a
	$36 = 2^2 \times 3^2$ (or 3 numbers that multiply to give 36 e.g. 2, 2, 9 etc)	dM1	2.1
	Since a, b and c are distinct must be **2, 3, 6** **($a = 2$, $b = 3$, $c = 6$)**	A1	3.2a
		(5)	
(b)	$(\log_{36} a)^2 + (\log_{36} b)^2 + (\log_{36} c)^2$	M1	3.4
	[= 0.0374137…+ 0.09398737…+0.25]		
	= 0.38140… awrt **0.381**	A1	1.1b
		(2)	
		(7 marks)	

	Notes
(a)	1st M1 for a start to the problem using sum of probabilities leading to eq'n in a, b and c
	1st A1 for reducing to the equation $abc = 36$ [Must follow from their equation.]
NB	Can go straight from $abc = 36$ to the answer for full marks for part (a).
	B1 for deducing that each value > 1 (may be implied by 3 integers all > 1 in the next line)
	2nd dM1 (dep on M1A1) for writing 36 as a product of prime factors or 3 values with product = 36 and none = 1
	2nd A1 for 2, 3 and 6 as a list or $a = 2$, $b = 3$ and $c = 6$
SC **Ans only**	**M0M0** If no method marks scored but a correct answer given score: M0A0B1M0A1 (2/5) This gets the SC score of 2/5 [Question says show your working clearly]
(b)	M1 for a correct expression in terms of a, b and c or values; ft their integers a, b and c Condone invisible brackets if the answer implies they are used.
	A1 for awrt 0.381