

Year 1 Pure Chapter 14: Differentiation and Modelling – Exam Questions (Total Marks 37)

1. The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y -coordinate of P is 8.

- (a) Find, in terms of $\ln 2$, the x -coordinate of P .

(2)

- (b) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found.

(4)

(Total 6 marks)

2. A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T °C, t minutes after it enters the liquid, is given by

$$T = 400 e^{-0.05t} + 25, t \geq 0.$$

- (a) Find the temperature of the ball as it enters the liquid. (1)
- (b) Find the value of t for which $T = 300$, giving your answer to 3 significant figures. (4)
- (c) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$. Give your answer in °C per minute to 3 significant figures. (3)
- (d) From the equation for temperature T in terms of t , given above, explain why the temperature of the ball can never fall to 20 °C. (1)

(Total 9 marks)

3. Rabbits were introduced onto an island. The number of rabbits, P , t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{3}t} \quad t \in \mathbb{R}, t \geq 0$$

- (a) Write down the number of rabbits that were introduced to the island. (1)

- (b) Find the number of years it would take for the number of rabbits to first exceed 1000. (2)

- (c) Find $\frac{dp}{dt}$. (2)

- (d) Find P when $\frac{dp}{dt} = 50$. (3)
- (Total 8 marks)**

4. Every £1 of money invested in a savings scheme continuously gains interest at a rate of 4% per year. Hence, after x years, the total value of an initial £1 investment is £ y , where

$$y = 1.04^x.$$

- (a) Sketch the graph of $y = 1.04^x$, $x \geq 0$. (2)
- (b) Calculate, to the nearest £, the total value of an initial £800 investment after 10 years. (2)
- (c) Use logarithms to find the number of years it takes to double the total value of any initial investment. (3)

(Total 7 marks)

5. The amount of a certain type of drug in the bloodstream t hours after it has been taken is given by the formula

$$x = De^{-\frac{1}{8}t},$$

where x is the amount of the drug in the bloodstream in milligrams and D is the dose given in milligrams.

A dose of 10 mg of the drug is given.

- (a) Find the amount of the drug in the bloodstream 5 hours after the dose is given.
Give your answer in mg to 3 decimal places.

(2)

A second dose of 10 mg is given after 5 hours.

- (b) Show that the amount of the drug in the bloodstream 1 hour after the second dose is 13.549 mg to 3 decimal places.

(2)

No more doses of the drug are given. At time T hours after the second dose is given, the amount of the drug in the bloodstream is 3 mg.

- (c) Find the value of T .

(3)

(Total 7 marks)