AS and A level Further Mathematics Practice Paper – Matrix algebra (part 1) – Mark scheme

Question	Scheme	Marks
1	$\mathbf{M} = \begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$	
	$\det\mathbf{M} = x(4x - 11) - (3x - 6)(x - 2)$	M1
	$x^2 + x - 12$ (=0)	A1
	$(x + 4)(x - 3)$ (= 0) $\rightarrow x = \ldots$	M1
	$x = -4,\ x = 3$	A1
		(4 marks)
2(a)	$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$, $\mathbf{P} = \begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix}$	
	$\mathbf{A}^{-1} = \frac{1}{10}\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$	M1 A1
		(2)
2(b)	$\mathbf{P} = \mathbf{AB}$	
	$\Rightarrow\ \mathbf{A}^{-1}\mathbf{P} = \mathbf{A}^{-1}\mathbf{AB}\ \Rightarrow\ \mathbf{B} = \mathbf{A}^{-1}\mathbf{P}$	
	$\mathbf{B} = \frac{1}{10}\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix}$	M1
	$= \begin{pmatrix} 2 & 1 \\ 1 & -4 \end{pmatrix}$	A1 A1
		(3)
		(5 marks)

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Question	Scheme	Marks
3(i)(a)	$\mathbf{A} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix}$, $\mathbf{B} = \mathbf{A} + 3\mathbf{I}$	
	$\mathbf{B} = \mathbf{A} + 3\mathbf{I} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix} + 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	M1
	$= \begin{pmatrix} 2k+4 & k \\ -3 & -2 \end{pmatrix}$	A1
		(2)
3(i)(b)	**B** is singular $\Rightarrow \det \mathbf{B} = 0$.	
	$-2(2k+4) - (-3k) = 0$	M1
	$-4k - 8 + 3k = 0$	
	$k = -8$	A1cao
		(2)
3(ii)	$\mathbf{C} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} 2 & -1 & 5 \end{pmatrix}$, $\mathbf{E} = \mathbf{CD}$	
	$\mathbf{E} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \begin{pmatrix} 2 & -1 & 5 \end{pmatrix} = \begin{pmatrix} 4 & -2 & 10 \\ -6 & 3 & -15 \\ 8 & -4 & 20 \end{pmatrix}$	M1
		A1
		(2)
		(6 marks)

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Question	Scheme	Marks
4(a)	$\mathbf{A} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix}$	
	$= \begin{pmatrix} 3+1+0 & 3+2-3 \\ 4+5+0 & 4+10-5 \end{pmatrix}$	M1
	$= \begin{pmatrix} 4 & 2 \\ 9 & 9 \end{pmatrix}$	A1
		(2)
4(b)	$\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix}$, where k is a constant,	
	$\mathbf{C} + \mathbf{D} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix} + \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix} = \begin{pmatrix} 8 & 2k+2 \\ 12 & 6+k \end{pmatrix}$	M1
	$\mathbf{E}$ does not have an inverse $\Rightarrow$ $\det \mathbf{E} = 0$.	
	$8(6+k) - 12(2k+2)$	M1
	$8(6+k) - 12(2k+2) = 0$	M1
	$48 + 8k = 24k + 24$ $24 = 16k$	
	$k = \frac{3}{2}$	A1 oe
		(4)
		(6 marks)

Question	Scheme		Marks
5(a)	$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$		
	$\mathbf{AB} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$		
	$= \begin{pmatrix} 2(-3)+0(5) & 2(-1)+0(2) \\ 5(-3)+3(5) & 5(-1)+3(2) \end{pmatrix}$	A correct method to multiply out two matrices. Can be implied by two out of four correct elements.	M1
	$= \begin{pmatrix} -6 & -2 \\ 0 & 1 \end{pmatrix}$	Any three elements correct	A1
		Correct answer	A1
		Correct answer only 3/3	**(3)**
5(b)	Reflection; about the y-axis.	Reflection	M1
		y-axis (or $x = 0$.)	A1
			(2)
5(c)	$\mathbf{C}^{100} = \mathbf{I} = \underline{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$	$\underline{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$ or $\mathbf{I}$	B1
			(1)
			(6 marks)

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Question	Scheme		Marks
6(a)(i)	$\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$		
	$\mathbf{A}^2 = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}\begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$		
	$= \begin{pmatrix} 1+2 & \sqrt{2}-\sqrt{2} \\ \sqrt{2}-\sqrt{2} & 2+1 \end{pmatrix}$	A correct method to multiply out two matrices. Can be implied by two out of four correct elements.	M1
	$= \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	Correct answer	A1
			(2)
6(a)(ii)	**Enlargement**; scale factor 3, centre $(0, 0)$.	**Enlargement**;	B1
		scale factor **3**, centre **(0, 0)**	B1
			(2)
6(b)	$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$		
	Reflection; in the line $y = -x$.	**Reflection**;	B1
		$y = -x$	B1
			(2)
6(c)	$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix}$, k is a constant.		
	C is singular $\Rightarrow \det \mathbf{C} = 0$. (Can be implied)	$\det \mathbf{C} = 0$	B1
	$9(k+1) - 12k \; (= 0)$	Applies $9(k+1) - 12k$	M1
	$9k + 9 = 12k$		
	$9 = 3k$		
	$k = 3$	$k = 3$	A1
			(3)
			(9 marks)

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Question	Scheme	Marks
7(i)	$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix}$	
	$\begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 5 & 6 \\ 5 & -6 & 11 \\ 13 & 11 & 21 \end{pmatrix}$	M1A2
	$\mathbf{BA} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 15 & 25 \\ 14 & 4 \end{pmatrix}$	B1
		(4)
7(ii)	$(\det \mathbf{C} =) 2k \times k - 3 \times (-2)$	M1
	$\mathbf{C}^{-1} = \dfrac{1}{2k^2 + 6}\begin{pmatrix} k & 2 \\ -3 & 2k \end{pmatrix}$	M1A1
		(3)
		(7 marks)
8(a)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B1
		(1)
8(b)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1
		(1)
8(c)	$\mathbf{R} = \mathbf{QP}$	B1
		(1)
8(d)	$\mathrm{R} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	M1 A1 cao
		(2)
8(e)	Reflection in the y axis	B1 B1
		(2)
		(7 marks)

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Question	Scheme		Marks
9(a)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 & 2 \\ 1 & 1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \end{pmatrix}$	Attempt to multiply the right way round with at least 4 correct elements	M1
	T' has coordinates (1,1), (1,2) and (4,2) or $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ **NOT just** $\begin{pmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \end{pmatrix}$	Correct coordinates or vectors	A1
			(2)
9(b)	**Reflection** in the line $\boldsymbol{y = x}$	Reflection $y = x$	B1 B1
	Allow 'in the axis' 'about the line' $y = x$ etc. Provided both features are mentioned ignore any reference to the origin unless there is a clear contradiction.		**(2)**
9(c)	$\mathbf{QR} = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$	2 correct elements Correct matrix	M1 A1
	Note that $\mathbf{RQ} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 10 & -4 \\ 24 & -10 \end{pmatrix}$ **scores M0A0 in (c) but allow all the marks in (d) and (e)**		**(2)**
9(d)	$\det(\mathbf{QR}) = -2 \times 2 - 0 = -4$	"-2"×"2" – "0"×"0"	M1
		-4	A1
	Answer only scores 2/2 $\frac{1}{\det(\mathbf{QR})}$ **scores M0**		**(2)**
9(e)	Area of $T = \frac{1}{2} \times 1 \times 3 = \frac{3}{2}$	Correct area for T	B1
	Area of $T'' = \frac{3}{2} \times 4 = 6$	Attempt at $"\frac{3}{2}" \times \pm "4"$	M1
		6 or follow through their det(QR) x Their triangle area provided area > 0	A1ft
			(3)
			(11 marks)

Question	Scheme	Marks
10(i)(a)	$\mathbf{A} = \begin{pmatrix} p & 2 \\ 3 & p \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -5 & 4 \\ 6 & -5 \end{pmatrix}$, $\mathbf{M} = \begin{pmatrix} a & -9 \\ 1 & 2 \end{pmatrix}$	
	$\{\mathbf{AB}\} = \begin{pmatrix} -5p+12 & 4p-10 \\ -15+6p & 12-5p \end{pmatrix}$	M1 A1
		(2)
10(i)(b)	$\{\mathbf{AB} + 2\mathbf{A} = k\mathbf{I}\}$	
	$\begin{pmatrix} -5p+12 & 4p-10 \\ -15+6p & 12-5p \end{pmatrix} + 2\begin{pmatrix} p & 2 \\ 3 & p \end{pmatrix} = k\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	
	$\begin{pmatrix} -3p+12 & 4p-6 \\ -9+6p & 12-3p \end{pmatrix} = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$	
	$"4p-10"+4=0$ or $"-15+6p"+6=0$ or $"-9+6p"="4p-6"$	M1
	$\Rightarrow p = \frac{3}{2}$	A1
	$k = -5\left(\frac{3}{2}\right)+12+2\left(\frac{3}{2}\right) \Rightarrow k = \ldots$	M1
	$k = \frac{15}{2}$	A1
		(4)
10(ii)	$\pm\frac{270}{15}$ $\{=\pm 18\}$	B1
	$\det\mathbf{M} = (a)(2)-(-9)(1)$	M1
	$\Rightarrow 2a+9=18$ or $2a+9=-18$	M1
	$\Rightarrow a=4.5$ or $a=-13.5$	A1 A1
		(4)
		(11 marks)

Question	Scheme		Marks
11(a)	$\mathbf{A} = \begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}$, where a and b are constants.		
	$\mathbf{A}\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$		
	Therefore, $\begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$	Using the information in the question to form the matrix equation. Can be implied by both correct equations below.	M1
	So, $-16 + 6a = 2$ and $4b - 12 = -8$	Any one correct equation.	
	Allow $\begin{pmatrix} -16+6a \\ 4b-12 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$	Any correct horizontal line	M1
	giving $a = 3$ and $b = 1$.	Any one of $a = 3$ or $b = 1$.	A1
		Both $a = 3$ and $b = 1$.	A1
			(4)
11(b)	$\det \mathbf{A} = 8 - (3)(1) = 5$	Finds determinant by applying $8 -$ their ab.	M1
		$\det \mathbf{A} = 5$	A1
	Area $S = (\det \mathbf{A})(\text{Area } R)$		
	Area $S = 5 \times 30 = 150 \text{ (units)}^2$	$\dfrac{30}{\text{their } \det \mathbf{A}}$ or $30 \times (\text{their } \det \mathbf{A})$	M1
		150 or ft answer	A1
			(4)
			(8 marks)

Question	Scheme	Marks
12(a)	**Rotation** ,135 degrees or $\frac{3\pi}{4}$radians (anticlockwise) about O or 225 degrees or $\frac{5\pi}{4}$ clockwise about O.	M1 A1 **(2)**
12(b)	$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 6\sqrt{2} \\ 3\sqrt{2} \end{pmatrix}$ $-p-q=12$ and $p-q=6$ or equivalent $p=-3$ and $q=-9$ or $\begin{pmatrix} -3 \\ -9 \end{pmatrix}$	 M1 A1 B1 cso **(3)**
12(c)	$\mathbf{Q} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	B1 **(1)**
12(d)	$\mathbf{R} = "\mathbf{Q}"\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$	M1 A1 A1 **(3)**
12(e)	$\mathbf{R}^{-1} = \frac{1}{-1}\begin{pmatrix} -\frac{1}{\sqrt{2}} & +\frac{1}{\sqrt{2}} \\ +\frac{1}{\sqrt{2}} & +\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$ **= R** (so matrix is self inverse and so transformation is self inverse)	 B1 **(1)**
		(10 marks)

Question	Scheme	Marks
13(a)	$\mathbf{A}+\mathbf{B}=\begin{pmatrix}1 & 2\\ -1 & 1\end{pmatrix}$	M1A1
	$\mathbf{2A}-\mathbf{B}=\begin{pmatrix}5 & 1\\ -2 & -1\end{pmatrix}$	M1A1
	$(\mathbf{A}+\mathbf{B})(\mathbf{2A}-\mathbf{B})=\begin{pmatrix}1 & 2\\ -1 & 1\end{pmatrix}\begin{pmatrix}5 & 1\\ -2 & -1\end{pmatrix}$	
	$\begin{pmatrix}1 & 2\\ -1 & 1\end{pmatrix}\begin{pmatrix}5 & 1\\ -2 & -1\end{pmatrix}=\begin{pmatrix}1 & -1\\ -7 & -2\end{pmatrix}$	M1A1
		(6)
13(b)	$\mathbf{MC}=\mathbf{A}\Rightarrow\mathbf{C}=\mathbf{M}^{-1}\mathbf{A}$	B1
	$\mathbf{M}^{-1}=\frac{1}{-2-7}\begin{pmatrix}-2 & 1\\ 7 & 1\end{pmatrix}$	M1
	$\mathbf{C}=\frac{1}{-2-7}\begin{pmatrix}-2 & 1\\ 7 & 1\end{pmatrix}\begin{pmatrix}2 & 1\\ -1 & 0\end{pmatrix}$	dM1
	$\mathbf{C}=-\frac{1}{9}\begin{pmatrix}-5 & -2\\ 13 & 7\end{pmatrix}$	A1
		(4)
		(10 marks)

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	Source paper	Question number	New spec references	Question description	New AOs
1	FP1 2013	1		Matrix algebra	1.1b, 2.1, 3.1a
2	FP1 2017	2		Matrix algebra	1.1b, 2.1, 3.1a
3	FP1 2013R	2		Matrix algebra	1.1b, 2.1, 3.1a
4	FP1 2012	2		Matrix algebra	1.1b, 2.1, 3.1a
5	FP1 2011	2		Matrix algebra	1.1b, 3.1a
6	FP1 2011	3		Matrix algebra	1.1b, 2.1, 3.1a
7	FP1 2014	4		Matrix algebra	1.1b, 2.4
8	FP1 Jan 2013	4		Matrix algebra	1.1b, 2.1, 3.1a
9	FP1 Jan 2012	4		Matrix algebra	1.1b, 2.1, 3.1a
10	FP1 2017	5		Matrix algebra	11.1.2, 13.1.1
11	FP1 2011	5		Matrix algebra	1.1b, 2.1, 3.1a
12	FP1 2016	6		Matrix algebra	1.1b, 2.1, 3.1a
13	FP1 2014R	6		Matrix algebra	1.1b, 3.1a