

①

$$2x + \frac{5x^3}{3} + C$$

②

$$\frac{8x^4}{4} + \frac{6x^{3/2}}{3/2} - 5x + C$$

$$2x^4 + \frac{12x^{3/2}}{3} - 5x + C$$

$$2x^4 + 4x^{3/2} - 5x + C$$

③

$$y = \frac{3x^3}{3} - \frac{3x^{3/2}}{3/2} - 7x + C$$

$$y = x^3 - \frac{6x^{3/2}}{3} - 7x + C$$

$$y = x^3 - 2x^{3/2} - 7x + C$$

$$y = x^3 - 2\sqrt{x}^3 - 7x + C$$

$$(4, 22)$$

$$22 = 4^3 - 2\sqrt{4}^3 - 7(4) + C$$

$$22 = 64 - 16 - 28 + C$$

$$C = 2$$

$$y = x^3 - 2\sqrt{x}^3 - 7x + 2$$

④

$$a) \quad \frac{dy}{dx} = \frac{(x^2+3)(x^2+3)}{x^2} = \frac{x^4 + 6x^2 + 9}{x^2}$$

$$= \frac{x^4}{x^2} + \frac{6x^2}{x^2} + \frac{9}{x^2}$$

$$= x^2 + 6 + 9x^{-2}$$

$$b) \quad y = \frac{x^3}{3} + 6x + \frac{9x^{-1}}{-1} + C$$

$$= \frac{x^3}{3} + 6x - \frac{9}{x} + C$$

$$(3, 20)$$

$$20 = \frac{3^3}{3} + 6(3) - \frac{9}{3} + C$$

$$20 = 9 + 18 - 3 + C$$

$$20 = 24 + C$$

$$C = -4$$

$$y = \frac{x^3}{3} + 6x - \frac{9}{x} - 4$$

⑤

a)

$$\frac{dy}{dx} = 3x - 5x^{-1/2} - 2$$

$$y = \frac{3x^2}{2} - \frac{5x^{1/2}}{1/2} - 2x + C$$

$$y = \frac{3x^2}{2} - 10x^{1/2} - 2x + C$$

$$y = \frac{3x^2}{2} - 10\sqrt{x} - 2x + C$$

$$(4, 5)$$

$$5 = \frac{3(4)^2}{2} - 10\sqrt{4} - 2(4) + C$$

$$5 = 24 - 20 - 8 + C$$

$$5 = -4 + C$$

$$C = 9$$

$$y = \frac{3}{2}x^2 - 10\sqrt{x} - 2x + 9$$

⑤

b)

$$\frac{dy}{dx} = 3x - \frac{5}{\sqrt{x}} - 2$$

$$(4, 5)$$

$$\frac{dy}{dx} = 3(4) - \frac{5}{\sqrt{4}} - 2$$

$$= 12 - \frac{5}{2} - 2$$

$$= 7.5$$

$$y = 7.5x + C$$

$$(4, 5)$$

$$5 = 7.5(4) + C$$

$$5 = 30 + C$$

$$C = -25$$

$$y = 7.5x - 25$$

(x2)

$$2y = 15x - 50$$

$$0 = 2y - 15x + 50$$

⑥

a)

$$f(x) = \frac{6x^3}{3} - \frac{10x^2}{2} - 12x + C$$

$$= 2x^3 - 5x^2 - 12x + C$$

$$(5, 65)$$

$$65 = 2(5)^3 - 5(5)^2 - 12(5) + C$$

$$65 = 250 - 125 - 60 + C$$

$$65 = 65 + C$$

$$C = 0$$

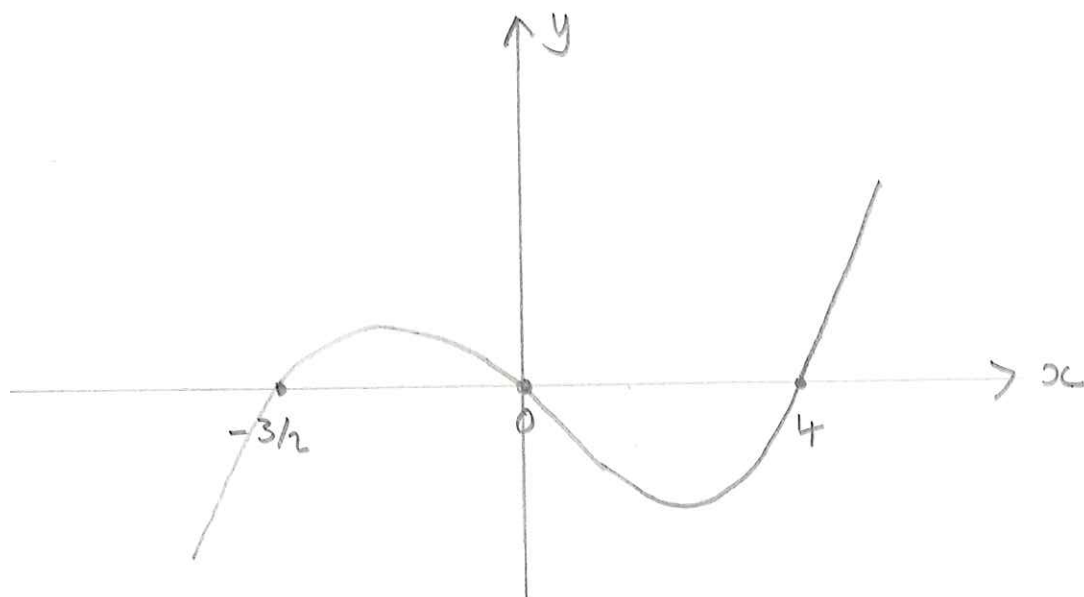
$$f(x) = 2x^3 - 5x^2 - 12x$$

b)

$$f(x) = x(2x^2 - 5x - 12)$$

$$= x(2x+3)(x-4)$$

c)



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a) $\frac{d^2y}{dx^2} = 3x^2 + 2$

b) $x^2 \geq 0$ hence $\frac{d^2y}{dx^2} \geq 0$

c) $y = \frac{x^4}{4} + \frac{2x^2}{2} - 7x + C$

$$y = \frac{x^4}{4} + x^2 - 7x + C$$

$$(2, 4)$$

$$4 = \frac{2^4}{4} + 2^2 - 7(2) + C$$

$$4 = 4 + 4 - 14 + C$$

$$4 = -6 + C$$

$$C = 10$$

$$y = \frac{x^4}{4} + x^2 - 7x + 10$$

⑦

$$\begin{aligned}d) \quad \frac{dy}{dx} &= x^3 + 2x - 7 \\&= 2^3 + 2(2) - 7 \\&= 8 + 4 - 7 \\&= 5\end{aligned}$$

$$\text{tangent gradient} = 5$$

$$\therefore \text{normal gradient} = -\frac{1}{5}$$

$$y = -\frac{1}{5}x + c$$

$$(2, 4)$$

$$4 = -\frac{1}{5}(2) + c$$

$$4 = -\frac{2}{5} + c$$

$$(\times 5)$$

$$20 = -2 + 5c$$

$$5c = 22$$

$$c = \frac{22}{5}$$

$$y = -\frac{1}{5}x + \frac{22}{5} \quad \text{hence}$$

$$\begin{aligned}5y &= -x + 22 \\5y + x - 22 &= 0\end{aligned}$$